Evaluation does not add or remove a global variable

For any $e, \xi, \phi, \rho, v, \xi',$ and $\rho'$ such that

$\langle e, \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle,$

we can prove

$\text{dom } \xi = \text{dom } \xi'$

“Evaluation doesn’t change the global domain”
Assume the existence of a derivation

Could terminate in any rule!

Base case:

\[ \langle \text{LITERAL}(v), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho \rangle \]

Both sides identical!

\[ \text{dom} \xi = \text{dom} \xi \]
Holds for formal-parameter lookup

Another base case:

\[ x \in \text{dom}\ \rho \]

\[ \langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle \]

Both sides identical!

\[ \text{dom}\ \xi = \text{dom}\ \xi \]
Inductive case: good sub-derivation

Assignment to formal parameter

\[
\begin{align*}
\text{By induction hypothesis on } \mathcal{D}, \quad \text{dom } \xi &= \text{dom } \xi' \\
\text{Both sides have same domain!}
\end{align*}
\]
Inductive case: good sub-derivation

True conditional

\[
\begin{array}{c}
\mathcal{D}_1 \\
\langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi', \phi, \rho' \rangle & \quad v_1 \neq 0 \\
\langle \text{IF}(e_1, e_2, e_3), \xi, \phi, \rho \rangle \Downarrow \langle v_2, \xi'', \phi, \rho'' \rangle
\end{array}
\]

By induction hypothesis on \( \mathcal{D}_1 \), \( \text{dom} \xi = \text{dom} \xi' \)

By induction hypothesis on \( \mathcal{D}_2 \), \( \text{dom} \xi' = \text{dom} \xi'' \)

Therefore, both sides have same domain:
\( \text{dom} \xi = \text{dom} \xi'' \)
The only interesting case: assign to global

\[
\begin{align*}
&x \notin \text{dom } \rho \quad x \in \text{dom } \xi \\
\frac{\langle e, \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \downarrow \langle v, \xi' \{x \mapsto v \}, \phi, \rho' \rangle}
\end{align*}
\]

Do both sides have same domain?

- **Does** $\text{dom } \xi = \text{dom } (\xi' \{x \mapsto v \})$?

By induction hypothesis on $\mathcal{D}$, $\text{dom } \xi = \text{dom } \xi'$

And $\text{dom } (\xi' \{x \mapsto v \}) = \text{dom } \xi' \cup \{x\} = \text{dom } \xi \cup \{x\}$

But $x \in \text{dom } \xi$! So $\text{dom } \xi \cup \{x\} = \text{dom } \xi$
And now: Scheme!
Examples of S-Expression operators

(cons 'a '()) also written '(a)

(cons 'b '(a)) equals '(b a)

(cons 'c '(b a)) equals '(c b a)

(null? '(c b a)) equals #f

(cdr '(c b a)) equals '(b a)

(car '(c b a)) equals 'c