Assignment 3

This assignment is due by the start of class on Monday, November 7.

1. Bayesian Inference for Univariate Normal: Consider an IID sample for a Normal variable

   \[ p(X|\mu, \lambda) = \prod_{i=1}^{N} \left( \frac{\lambda}{2\pi} \right)^{1/2} e^{-\frac{\lambda}{2}(x_i-\mu)^2} \]

   with mean \( \mu \) and precision \( \lambda \), and consider a Normal-Gamma prior given by

   \[ p(\mu, \lambda) = \mathcal{N}(\mu|\mu_0, \frac{1}{\beta_0\lambda}) \text{ Gamma}(\lambda|a_0, b_0) \].

   Calculate the posterior for \( \mu \) and \( \lambda \) and show that it is also decomposable as a Normal-Gamma distribution, with new parameters \( a_N, b_N, \beta_N, \mu_N \). In particular explicitly calculate and show the parameters of the posterior when given in this form.

   Note: the text on page 101 of the textbook is relevant for this question but the notation there can be confusing. It is therefore recommended to use the notation given here.

2. Solve problem 4.11 (page 222) in the textbook.

3. Calculate one iteration of the Newton-Raphson method for minimizing the function \( f(x) = x_1^3 + 5x_1x_2^2 - 7x_1^2x_2 \) where \( x = (x_1, x_2)^T \). Use \( x = (1, 1)^T \) as the initial value.

4. Consider a Poisson likelihood function, \( \text{Poisson}(x|\lambda) = \frac{1}{x!} e^{-\lambda} \lambda^x \), with prior, \( \text{Gamma}(\lambda|a, b) \), on \( \lambda \). Develop a Gaussian approximation to \( p(\lambda|x, a, b) \) using the Laplace approximation. First develop the formula in general and then apply it to the case \( a = 1, b = 1, x = 3 \). For this case, plot the approximation and the true function for \( 0 \leq \lambda \leq 10 \).

5. Solve problem 4.18 (page 223) in the textbook.