A.I. in health informatics
lecture 4 computational models for clinical decision making

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today

- tie many of the topics covered thus far together by studying computational systems for clinical reasoning

- review the Ledley & Lusted system; maybe some recent Bayes net stuff

Ledley & Lusted

- logically reasoning about diagnoses
- a system that could theoretically be implemented
  - from ‘59!
- a symbolic-logicish approach, so we’ll review that first

a bit of logic

- *propositions* are the basic units of logic
  - cannot be be broken down any further
  - “Kevin is a human being”

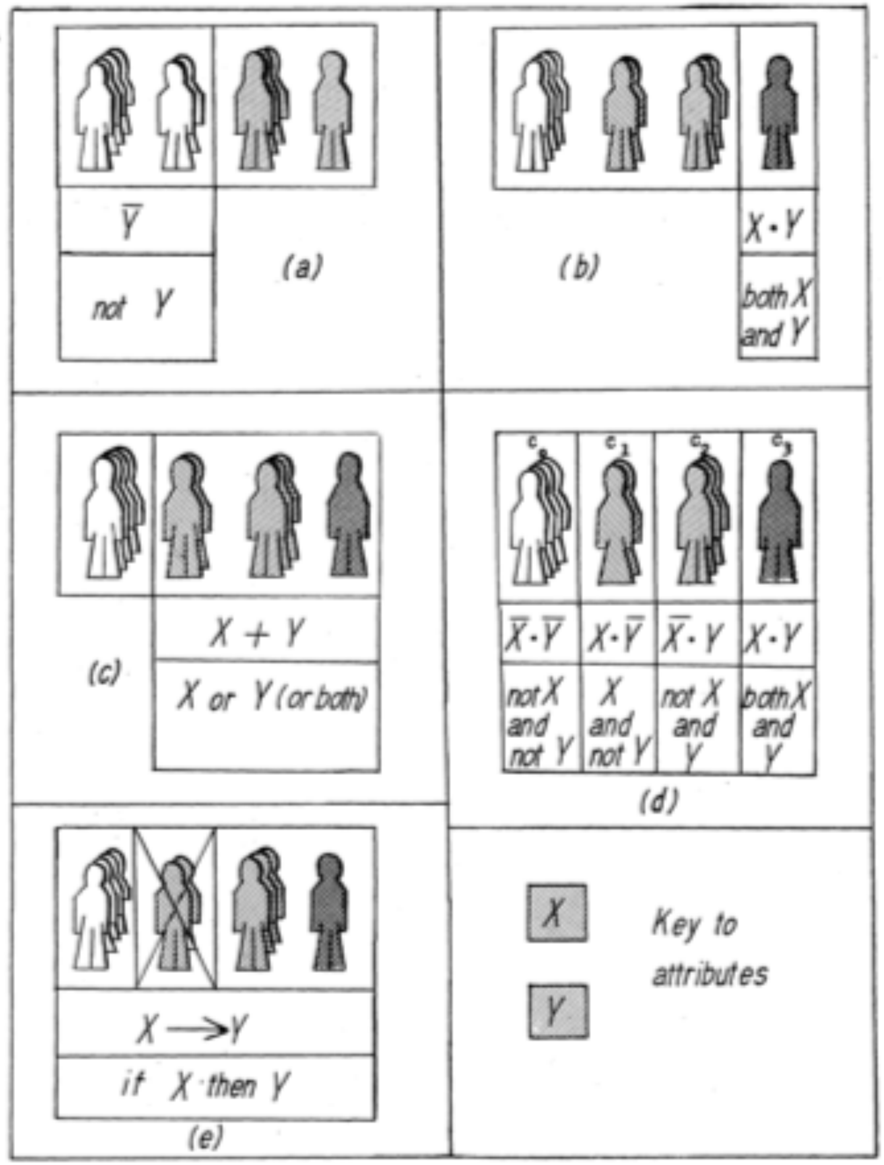
- *rules* operate on propositions

symbols & their meaning

Symbols	Name	Interpretation
$\overline{Y}$	Negation	Not Y
$X \cdot Y$	Logical product	X [illegible] Y
$X + Y$	Logical sum	[illegible]
$X \rightarrow Y$	Implies	[illegible]

this is all we need for the proposed system!

truth table refresher
(on board)



$\overline{Y}$
not Y
(a)
(b)
$X \cdot Y$
both X and Y
$X + Y$
X or Y (or both)
(c)
c_0
c_1
c_2
c_3
$\overline{X} \cdot \overline{Y}$
$X \cdot \overline{Y}$
$\overline{X} \cdot Y$
$X \cdot Y$
not X and not Y
X and not Y
not X and Y
both X and Y
(d)
$X \rightarrow Y$
if X then Y
(e)
X
Y
Key to attributes

dealing with symptoms & diseases; logically

symptoms $S_1, S_2 \ldots S_n$

diseases $D_1, D_2 \ldots D_m$

all of medical knowledge is some function:

$$E(S_1, S_2 \ldots S_n, D_1, D_2 \ldots D_m)$$

dealing with symptoms & diseases; logically

a *diagnosis* maps symptoms to diseases:

$$f(S_1, S_2 \ldots S_n) \rightarrow (D_1, D_2 \ldots D_k)$$

the evidence: logically stated

$$
\begin{aligned}
E = {} & [D(2) \rightarrow S(1)] \cdot \\
& [D(1) \cdot \overline{D(2)} \rightarrow S(2)] \cdot \\
& [\overline{D(1)} \cdot D(2) \rightarrow \overline{S(2)}] \cdot \\
& [S(1) + S(2) \rightarrow D(1) + D(2)]
\end{aligned}
$$

what does this say?

translation into symbolic logic

If a patient has disease 2, he must have symptom 1	$D(2) \rightarrow S(1)$
If a patient has disease 1 and not disease 2, then he must have symptom 2	$D(1) \cdot \overline{D(2)} \rightarrow S(2)$
If a patient has disease 2 and not disease 1, then he cannot have symptom 2	$\overline{D(1)} \cdot D(2) \rightarrow \overline{S(2)}$
If a patient has either or both of the symptoms, then he must have one or both of the diseases	$S(1) + S(2) \rightarrow D(1) + D(2)$

diagnosis

- so that's our evidence base

- suppose a patient has symptoms S_1 and $\overline{S_2}$
  - ie., not symptom 1, but symptom 2
  - note that we explicitly encode knowledge about what symptoms the patient does *not* have

deducing disease from symptoms

- our diagnosis $f(\overline{S_1}, S_2, E)$ is:

$$f(S_1 \text{ and } \overline{S_2}) = D_1\overline{D_2}$$

- how did we get that?
  - logical deduction!

deducing disease from symptoms

- remember we need to satisfy:

$$E \longrightarrow (G \longrightarrow f)$$

- in words:

if the medical evidence is *E*, then: if the symptoms are G then the disease is *f*

logical basis

the enumeration of all possible disease-symptom states

logical operations are performed over this space

logical basis example

two symptoms: S_1 and S_2
one disease D

S_1	0	0	0	0	1	1	1	1
S_2	0	0	1	1	0	0	1	1
D	0	1	0	1	0	1	0	1

all possible states -- but some of these are precluded by *E*

diagnosis by elimination

- two symptoms, two diseases

	C_0				C_1				C_2				C_3			
	C^0	C^1	C^2	C^3	C^0	C^1	C^2	C^3	C^0	C^1	C^2	C^3	C^0	C^1	C^2	C^3
$S(1)$	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$S(2)$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
$D(1)$	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
$D(2)$	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

patient has symptom 1 but not 2 and disease 2 but not 1

deductive diagnosis

E

$$\boxed{D(2) \rightarrow S(1)}$$

$$D(1) \cdot \overline{D(2)} \rightarrow S(2)$$

$$\overline{D(1)} \cdot D(2) \rightarrow \overline{S(2)}$$

$$S(1) + S(2) \rightarrow D(1) + D(2)$$

	C_0				C_1				C_2				C_3			
	c^0	c^1	c^2	c^3	c^0	c^1	c^2	c^3	c^0	c^1	c^2	c^3	c^0	c^1	c^2	c^3
S(1)	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
S(2)	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
D(1)	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
D(2)	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

D_2 implies S_1; thus the highlighted rows are eliminated

deductive diagnosis

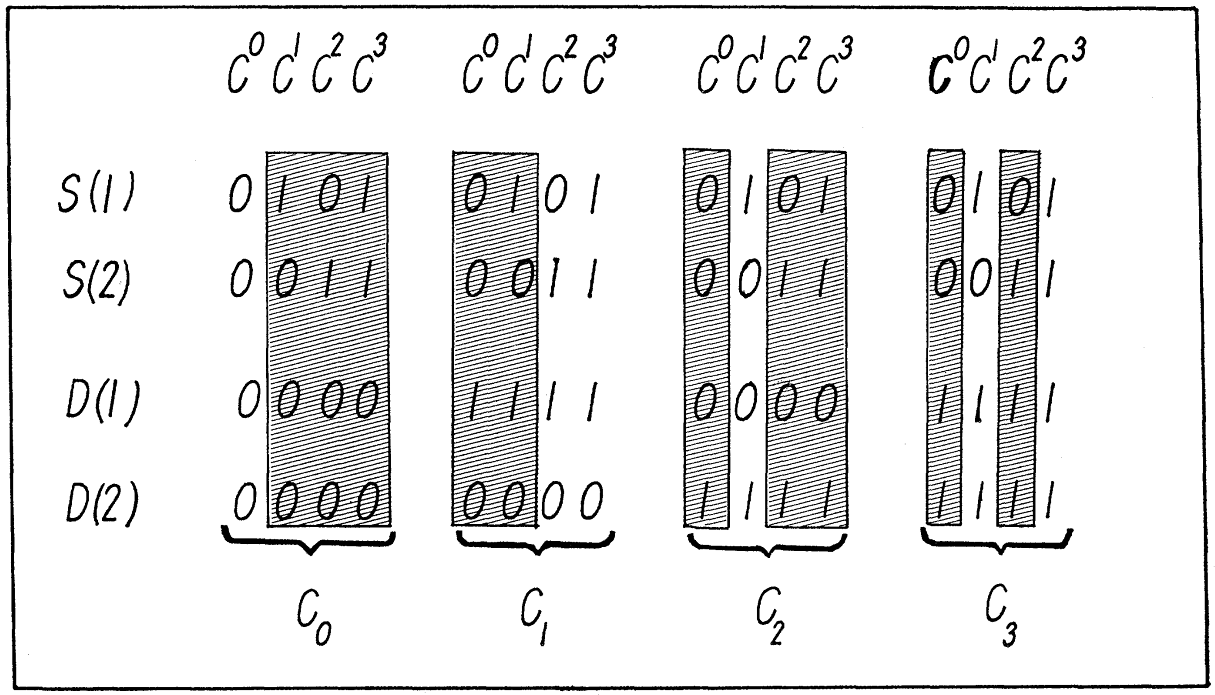
reduced basis (after applying all knowledge in *E*)

$$D(2) \rightarrow S(1)$$

$$D(1) \cdot \overline{D(2)} \rightarrow S(2)$$

$$\overline{D(1)} \cdot D(2) \rightarrow \overline{S(2)}$$

$$S(1) + S(2) \rightarrow D(1) + D(2)$$



deductive diagnosis

- recall that our patient has symptoms S_1 and $\overline{S_2}$
- using logical elimination, we can infer that the patient has $D_1\overline{D_2}$

deductive diagnosis

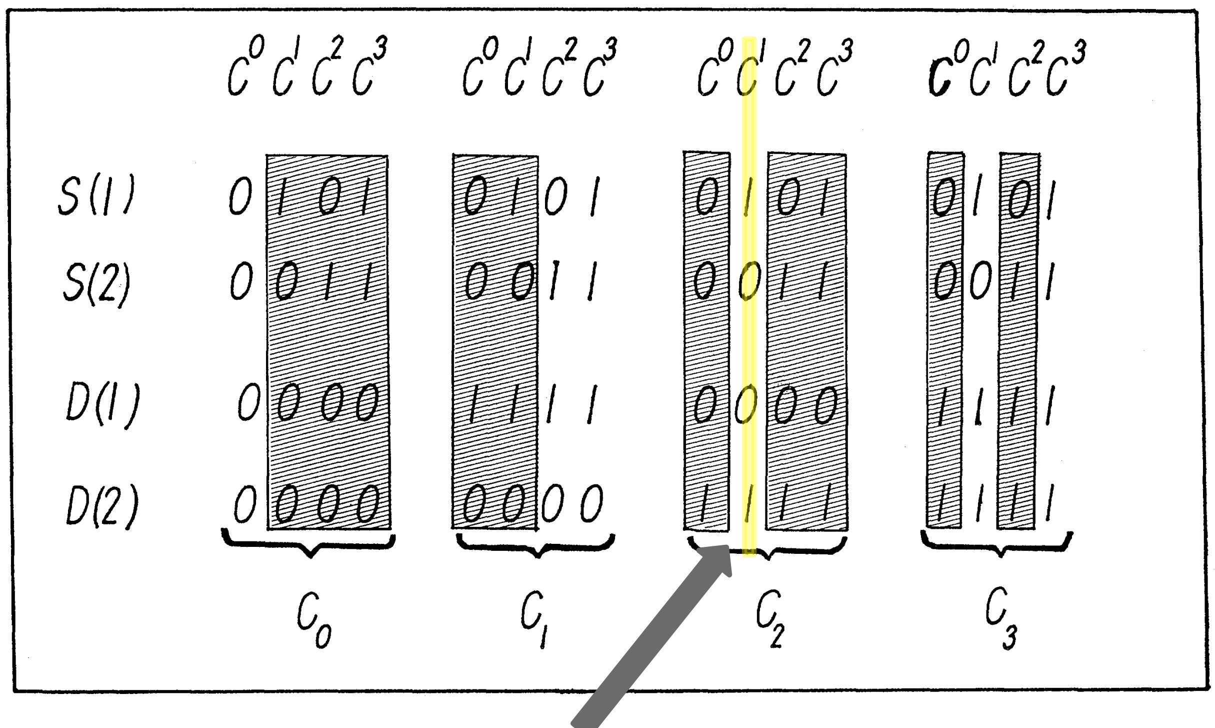
known: patient has S_1, not S_2

$$D(2) \rightarrow S(1)$$

$$D(1) \cdot \overline{D(2)} \rightarrow S(2)$$

$$\overline{D(1)} \cdot D(2) \rightarrow \overline{S(2)}$$

$$S(1) + S(2) \rightarrow D(1) + D(2)$$



it’s the only column consistent with symptoms and *E!*

problems with this approach

- relies strictly on Boolean inference
  - no room for uncertainty: want to say if a patient has S_1, S_2 then they have D_1 with some probability
  - solution: incorporate probabilistic reason
  - similar to what we discussed in Lecture 2

probabilistic logical reasoning

- now assume *E* is given in the form $P(C^k|C_i)$: probability of having symptoms given diseases

- turn this around with Bayes' theorem to make our diagnosis *f*

$$P(C_i|C^k) = \frac{P(C_i)P(C^k|C_i)}{\sum_{\omega} P(C_\omega)P(C^k|C_\omega)}$$

probabilistic logical reasoning

- evidence *E* provides the required conditional probabilities, presumably empirically gathered

$P(C^0 \vert C_0) = 1$	$P(C^1 \vert C_0) = 0$	$P(C^2 \vert C_0) = 0$	$P(C^3 \vert C_0) = 0$	$P(C_0) = 910/1000$
$P(C^0 \vert C_1) = 0$	$P(C^1 \vert C_1) = 0$	$P(C^2 \vert C_1) = 3/5$	$P(C^3 \vert C_1) =$ [illegible]	$P(C_1) = 50/1000$
$P(C^0 \vert C_2) = 0$	$P(C^1 \vert C_2) = 1$	$P(C^2 \vert C_2) = 0$	$P(C^3 \vert C_2)$ [illegible]	[illegible] 000
$P(C^0 \vert C_3) = 0$	$P(C^1 \vert C_3) = 2/3$	$P(C^2 \vert C_3) = 0$	$P(C^3 \vert C$ [illegible]	[illegible] 000

- note that
  - probability of symptoms co
    complexes
  - probability of logically preclud 0
  - we also know disease prevalenc

if we have *m* symptoms and *n* diseases we need $O(2^{n+m})$ entries! (Ledley kept the example short for a reason …

probabilistic logical reasoning

- the punch-line: given probabilities, we can select between two competing logically permissible diagnoses by assessing which is more probable
  - (see example in article for calculation; it’s just plug & chug from the given formula)

- OK. but how to select *treatments*?
  - utility theory!

putting it together

- medical evidence, *E*, either propositional or probabilistic (or mixed) is provided to our model

- given a set of symptoms, the *admissible* diagnoses (given *E*) are deduced using logic

- the diagnosis is taken as the most likely of these

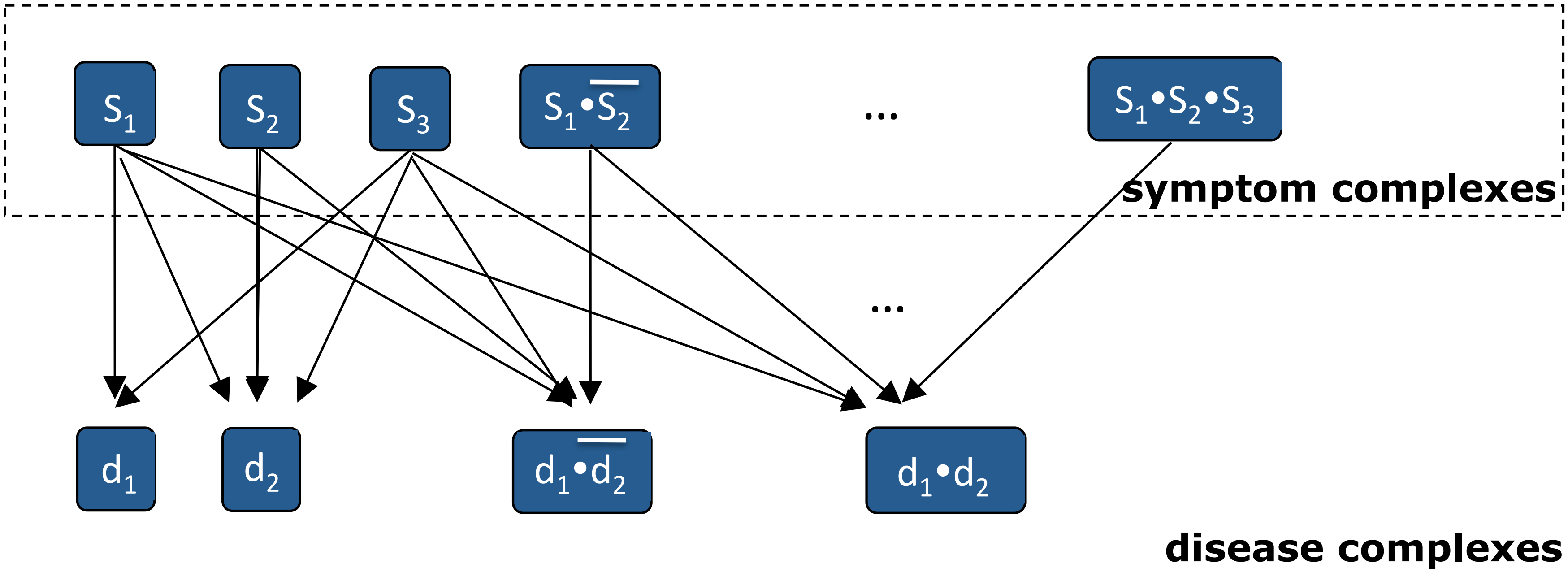
- the treatment is selected to maximize utility

relation to Bayes nets

- **Ledley & Lusted** proposed deductive symbolic logic based approach
  - the probabilistic version could be represented as a Bayes' net

- the general framework is the same: the question is: **how to encode the evidence** *E*?

Ledley & Lusted as a really simple Bayes net



- too many parameters to estimate!
- … maybe for small networks

DXplain

- comprises database of crude probabilities of over 4500 findings associated with 2000 different diseases
- uses Bayesian reasoning
- used by a number of hospitals and medical schools, mostly for educational purposes but also for clinical consultation
- http://dxplain.org/dxp/dxp.pl