COMP163 Homework Assignment 3: Due Wednesday, October 17, 2018 @ 4:30pm

Reading: Read the pertinent lecture notes and handouts. Also read section 6.5 (pp. 143-144) and then other parts of Chapter 6 if interested. Begin reading the lecture notes on Voronoi Diagrams as well as Chapter 7 in the text as we will cover this next week.

General Information: When describing an algorithm, do not forget to analyse its running time and explain why the algorithm is correct. Although you may discuss these problems in the preliminary stages with others, work submitted should be done individually. If you have any discussions with others (students, friends, TAs, faculty, ...) relative to a homework problem or if you gain information from a written source other than your own notes from lecture, you are expected to identify your collaborator/source.

Problems

1. **Mininum-Area Triangle**: Given a set of n points in the plane, the goal is to identify the minimum area triangle defined on those n points.
   (a) Give a naive $O(n^3)$ algorithm to compute the minimum-area triangle.
   (b) Given two points p_1 and p_2 and the line they define, describe how to order the remaining $n-2$ points in increasing order of the area of the triangle each forms with $p_1\bar{p}_2$. (Include a figure). How can you characterize the point which forms the smallest area triangle with p_1 and p_2?
   (c) Use duality and line sweep to create an algorithm for the minimum area triangle problem that runs in time $o(n^3)$.
   (d) START THINKING ABOUT THIS–we will discuss topological sweep further in class on 10/16 Can you then replace line sweep with topological line sweep and improve the running time? If yes, explain how you would do it and the final time complexity. If not, argue why not.

2. **Diameter and width [O'Rourke]**: Define the *diameter* of a set of points to be the largest distance between any two points in the set.
   a. Prove that the diameter of a set is achieved by two vertices of the convex hull of the set.
   b. A *line of support* to a set is a line L that touches the hull and has all points on or to one side of L. Prove that the diameter of a set is the same as the maximum distance between parallel lines of support for the set.
   c. Two points a and b are called *antipodal* if they admit parallel lines of support: there are parallel lines of support through a and b. Develop an algorithm for enumerating (listing) all antipodal pairs of a set of points in two dimensions. Verify its correctness and analyse its complexity.
   d. Define the *width* as the minimum distance between parallel lines of support. Develop an algorithm for computing the width of a set of points in two dimensions. Verify its correctness and analyse its complexity.

3. **Double Wedges**: Given a set S of n non-vertical line segments specified by the coordinates of their left-endpoints and right endpoints, and given a special line segment t, also specified by its left endpoint and its right endpoint, the goal is to determine whether there is an infinite line that intersects t but does **not** intersection any of the other segments in S, and, if so, give an equation of one such line.

   First briefly discuss what you might do to solve this problem in the primal. Then provide the best algorithm that you can to solve this problem in the dual, justify correctness, and analyse complexity.

4. **Your Programming Project** Begin thinking about what geometric algorithm/data structure you would like to implement OR what theoretical topic that you might want to investigate. Please send email to me as to your thoughts on this.