How would you define triangulation of a point set?

- Decomposition/partition of convex hull (into triangles!)
- Get triangular faces
- Edges only between points; no crossings
  - And no points leftover

How would you triangulate? How fast are we hoping for?

- Constructive: Insert edges while possible (proof?)
  - How many edges might we insert?
- Start with convex hull, maintain a weakly simple polygon
  - Add edges to new points
- Incremental/online
$V - E + F = 2$

* $F$ includes outer-face
applied to graph
w triangular outer face:
\[ V - E + F = 2 \]
\[ (n+3)(e+h+6)(f+h+3) \]

augment given triangulation:
(h = points on hull)

* F includes outer face
* E = \( \frac{3F}{2} \) \rightarrow 3 edges/\( \Delta \)

\[ V - E + \frac{2}{3}E = 2 \]
\[ V - 2 = \frac{1}{3}E \]
\[ E = 3V - 6 \]

\[ (e+h+6) = (3n+9) - 6 \]
\[ e = 3n - h - 3 \]

given:
n blue points
e pink edges
f triangular faces
Triangulation by creating a weakly simple polygon

Start with C.H.
Sweep to connect one point
sweep again... anywhere
Now we have a weakly simple polygon.

Triangulate
Assume hull = triangle

Start with $V = E = 3$

Incremental triangulation
Start with $V=E=3$

add +1 +3 each time

Join triangle to whatever random point you select inside
Start with $V=E=3$
add $+1 +3$ each time
shown in parallel

More arbitrary points inside
Start with $V=E=3$
add +1 +3 each time
If hull > triangle split problem

Can also do arbitrary incremental (online)

last added

How much time did these methods take?
the ART GALLERY (GUARDING) PROBLEM

- $\rightarrow$ a guard can see to $\infty$
  (but not through walls)
  in all directions

- $\rightarrow$ walls covered with $\&$ art

place fixed guards s.t. all art is seen
the ART GALLERY (GUARDING) PROBLEM

- → a guard can see to ∞ (but not through walls) in all directions
- → walls covered with art

place fixed guards s.t. all art is seen

visibility region of top guard leaves two gaps
the ART GALLERY (GUARDING) PROBLEM

• ➔ a guard can see to ∞ (but not through walls) in all directions

→ walls covered with $\text{art}$

place fixed guards s.t. all art is seen

one gap covered

still two gaps

top guard was necessary
the ART GALLERY (GUARDING) PROBLEM

- → a guard can see to oo (but not through walls) in all directions

→ walls covered with $\&$ art

place fixed guards s.t. all art is seen

last guard suffices
Given an n-gon art gallery

How many guards might we need?

You need a guard in this region

and it won’t help you guard any of the others

\( \frac{n}{3} \)
Can we match the $\left\lfloor \frac{n}{3} \right\rfloor$ bound
or are even more guards sometimes necessary?
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How many coins of each color?

\[ \bullet + \bullet + \bullet = n \]

8 \quad 9 \quad 9

\[ \leq \frac{n}{3} \text{ for } \geq 1 \text{ color} \]

...And...

every wall sees that color

DONE
Notice we placed guards on vertices

\[ \text{If obviously this suffices for matching } \lceil \frac{n}{3} \rceil \]

\[ \text{what if we had an "easily guardable" polygon?} \]

\[ \text{Can interior guards help more?} \]

\[ \text{yes} \]

(star-shaped)
What if I can afford a few more guards, and I don't want to spend time triangulating?

- A guard on every vertex: OK
- A guard on every convex vertex: No