MELKMAN'S ALGORITHM

- Note: not the first O(n) C.H. algo, but it is the simplest.
- It is an online algorithm: at any time you have the C.H. of the current input. No pre-scanning.

- Start w/ 3 points
- Store them in a DEQUE:
  \[
  \begin{pmatrix}
  3 \\
  2 \\
  1 \\
  3 \\
  \end{pmatrix}
  \]
- WLOG let 123 be \( \rightarrow \)
  \( \rightarrow \) then: bottom of deque: 2, 1, 3 = \( \uparrow \)
  top of deque: 1, 2, 3 = \( \downarrow \)

- Where could the next vertex be?
Inside region (yellow)

Do nothing

Also ignore the rest of the chain until some edge crosses 1-3

If this happens, we go to green or blue

(no difference from arriving in green/blue directly from 3)
Right region (green)

\[
\begin{bmatrix}
3 \\
2 \\
1 \\
3
\end{bmatrix} \times \begin{bmatrix}
4 \\
3 \\
2 \\
1 \\
4
\end{bmatrix}
\]

re-established C.H. in both directions

top of deque ok
bottom: last 3 not 8
Green is still the "right region"
Red is still "left region"
Always with respect to last point AND current hull.

1 should be deleted entirely
& 3 should still be deleted
Top of deque: 1, 2, 3, 4 : not C
2 must be removed
3 must be discarded entirely
Red: will pop 11 from top. Could pop 9, 3, 2, not 1.

Green: will pop 11 from bottom. Could pop 1, 2, not 3, 9

Blue: will pop 11 from top & bottom. Could pop 9 and/or 1.
Some new problems

- Find max dist among pairs of points in a set
- Find min. perimeter / area enclosing box
- Find max dist between red points and green points
Diameter of point set

max dist among any pair

- Obviously you can compute this in $O(n^2)$ time. Better?
- Only C.H. points are candidates. Why?
So diameter takes $O(h^2)$. 

What if we compute hull: $O(n \log h)$, then binary search for each point $L \rightarrow h \cdot \log h$

Find max dist "anchored" at each point

Distance is not monotonic
• = assume they define diameter

= LUNE

All points must be inside (not arbitrarily, but it's necessary)

So, 2 parallel lines on • must enclose all points

algorithm? time?
$O(n)$ edges = comparisons
IMPLEMENTATION: 2 forward scans on cyclic list (in parallel) $O(h)$ after $O(n \log h)$ sorting
Diameter of Polygon

\[ \text{max dist among any 2 pts on boundary} \] : candidates?

vertex-edge?

edge-edge?

only consider C.H. vertices : So \(O(h)\) time

(once you have the hull)
DIAMETER OF A CLOSED CURVE

Define curve of constant width

space shuttle ?
WIDTH OF POINT SET

NOT min dist among any pair
we want min width slab that contains the set
\[ \leftrightarrow \text{trivially, use rotating calipers} \]

The width is always determined by 3 points
\[ \leftrightarrow \text{easy unimodal argument when only 2 points} \]
MINIMUM ENCLOSED BOX

- area
- perimeter
where should we stop?
UNION OF 2
CONVEX POLYGONS

Same principle: merge 2 C.H.

Next edge:
compare angles:
current red - next blue
vs
current red - next red

current red →
current blue →
Union of 2 Convex Polygons

Same principle: merge 2 c.h.

The 2 calipers will meet on a hull edge and exchange positions.