BASIC OPERATIONS

Do ABC form a left turn?

Is P above L?
Do ABC form a left turn?
Do ABC form a left turn?

\[ \vec{v}_1 \times \vec{v}_2 \]

- \( > 0 \) \Rightarrow \text{left}
- \( < 0 \) \Rightarrow \text{right}
- \( = 0 \) \Rightarrow \text{collinear}
Is \( P \) above \( \text{L} \) ?  

\text{(left of)}

\[
\left\{ \right. 
\right. 
\text{same as before}
\]
Do 2 segments intersect?
Do 2 segments intersect?

Let \( \vec{r} = \overrightarrow{R_1R_2} \)

Then if \( B_2 \) is above \( \vec{r} \), \( B_1 \) must be below.

\( (R_1R_2B_2 : \text{left} \quad \& \quad R_1R_2B_1 : \text{right}) \)
Do 2 segments intersect?

Let \( \vec{r} = \overrightarrow{R_1R_2} \)

Then if \( B_2 \) is above \( \vec{r} \), \( B_1 \) must be below.

Without loss of generality

\((R_1 R_2 \overrightarrow{B_2} : \text{left} \& \overrightarrow{R_1R_2B_1} : \text{right})\)

Also, independently, compare \( R_1 \& R_2 \) w/ \( \vec{b} = \overrightarrow{B_1B_2} \)
Segment vs ray

wlog ray points right
Segment vs ray

1) verify that 2 endpoints are on opposite sides of
2) Look at \( \overrightarrow{CA} \overrightarrow{BC} \)

wlog ray points right
Point in Triangle
BASIC STRUCTURES

POINTS!

NOT IN GENERAL POSITION
Polygons
POLYGONS

SIMPLE

NON-SIMPLE
POLYGONS

SIMPLE

NON-SIMPLE

WEAKLY SIMPLE
Quick rule to decide if convex?
MONOTONE

ORTHOGONAL

STAR-SHAPED

etc
JORDAN CURVE theorem

\sim \text{ any closed curve } C
JORDAN CURVE theorem

any closed curve $C$

separates the plane in
2 components

$\leq 1$ bounded

$\geq 1$ unbounded
**JORDAN CURVE theorem**

Any closed curve $C$ separates the plane into two components: one bounded and one unbounded. And any path linking two points in different components must cross $C$. 
JORDAN CURVE theorem

\( \sim \) any closed curve \( C \)
separates the plane in
2 components \( \leq 1 \) bounded
AND
any path linking 2 points
in different components
must cross \( C \).
JORDAN CURVE theorem

~ any closed curve $C$

separates the plane in
2 components $< 1$ bounded
$\geq 1$ unbounded

AND

any path linking 2 points
in different components
must cross $C$. 
JORDAN CURVE theorem

Any closed curve $C$ separates the plane into 2 components: 1 bounded and 1 unbounded. AND any path linking 2 points in different components must cross $C$. 

Same Component?
The J.C. thm. has a relatively short proof for polygons. It is much worse for curves, especially non-piecewise differentiable. (e.g. Koch snowflake)
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Any 2 pts in same component are linked by a path that does not cross $\mathcal{C}$. A path can continuously shrink to a point...
The J.C. thm. has a relatively short proof for polygons. It is much worse for curves, especially non-piecewise differentiable. (e.g. Koch snowflake)

Any 2 pts in same component are linked by a path that does not cross $\mathcal{C}$. $\Rightarrow$ path can continuously shrink to a point...

In 3D, take a closed sphere, or other shape w/ same genus. $\Rightarrow$ any loop inside can shrink to a point $\Rightarrow$ but something strange can happen outside (horned sphere)
Back to testing: point in polygon?
Back to testing: point in polygon?

Easy: by Jordan
Back to testing: point in polygon

Harder?
Back to testing: point in polygon

Plumb line algorithm

- take ray to \( \infty \)
- iff odd # intersections : IN time?
WINDING NUMBER ALGORITHM

> start w/ arbitrary ray from ⬤ to a point on poly.
WINDING NUMBER ALGORITHM

> start w/ arbitrary ray from to a point on poly.
> rotate as you walk on poly.
WINDING NUMBER ALGORITHM

> start w/ arbitrary ray from \( \bullet \) to a point on poly.
> rotate as you walk on poly.
> count # full turns
> result: \[ \begin{cases} 1 & : \text{IN} \\ 0 & : \text{OUT} \end{cases} \]
Winding Number Algorithm

- Start with an arbitrary ray from \( \bullet \) to a point on poly.
- Rotate as you walk on poly.
- Count the number of full turns.
- Result: \( \{1 : \text{IN}, 0 : \text{OUT}\} \)
AREA of POLYGON
AREA of POLYGON

Could decompose into “easy” pieces

Involves constructing new (Steiner) vertices

Time?
Area of convex polygon

How would you do it?

How is the polygon represented?
Area of convex polygon

\[ \sim \text{Construct segment from } \bullet \text{ to every vertex } P_i \]
& add area of \( \Delta(P_iP_{i+1}) \)
Clearly doesn't work for non-convex

> counting area outside
> double counting area inside

\[ \text{triple, etc.} \]
Look at edges 1, 2, 3.

1: \[\rightarrow\] add
2: \[\rightarrow\] subtract
3: \[\rightarrow\] add

what happens to the area outside from \(\Delta_1\)?
what happens to the triple-counted area?
Proof:

SPLIT INTO RADIAL SECTORS

by one ray per vertex
Proof:

SPLIT INTO RADIAL SECTORS

Upper ray per vertex

Topmost segment is
& contributes positively

out

out

out
Proof:

SPLIT INTO RADIAL SECTORS

by one ray per vertex

Topmost segment is
& contributes positively

Next is & contributes neg.
Proof:

SPLIT INTO RADIAL SECTORS

by one ray per vertex

Topmost segment is & contributes positively

Next is & contributes neg.

So top 'inside-component' is counted properly, & the exterior below is 'cancelled out.' Actually these 2 Δ's cancel entirely, below.

... repeat

(same type of argument for winding number method)