Any diagonal splits a polygon into 2 chains. How balanced can this split be?

1. Can you find a diagonal to split $\frac{n}{2} \times \frac{n}{2}$? $\frac{9n}{10} \times \frac{n}{10}$?

2. What if I already give you a triangulation?
Finding a $\frac{n}{4} \vee \frac{3n}{4}$ split
given a triangulation

- Arbitrarily split polygon into 4 chains of size $\sim \frac{n}{4}$
Finding a $\frac{n}{4} \sqrt{\frac{3n}{4}}$ split given a triangulation

- Arbitrarily split polygon into 4 chains of size $\sim \frac{n}{4}$
- If you find a diagonal between $C_1 - C_3$ or $C_2 - C_4$ then done.
Finding a $\frac{n}{4} \sqrt{3n} \frac{3n}{4}$ split given a triangulation

- Arbitrarily split polygon into 4 chains of size $\sim \frac{n}{4}$
- If you find a diagonal between $C_1$-$C_3$ or $C_2$-$C_4$ then done.
- Not all diagonals can be within their chain. E.g., $C_1$-$C_1$ or $C_4$-$C_4$.
  Why? Because you would get a quad leftover.
Finding a $\frac{n}{4} \sqrt{3n^2} \text{ SPLIT GIVEN A TRIANGULATION}$

- Arbitrarily split polygon into 4 chains of size $\sim \frac{n}{4}$

- If you find a diagonal between $C_1-C_3$ or $C_2-C_4$ then Done.

- Not all diagonals can be within their chain e.g. $C_1-C_1$ or $C_4-C_4$.
  Why? Because you would get a quad leftover.

- So, there exists a diagonal from $C_1-C_2$, wlog
Finding a $\frac{n}{4} v \frac{3n}{4}$ split:

...cont.

- there exists a diagonal from $C_1 - C_2$
Finding a \( \frac{n}{4} \text{ vs } \frac{3n}{4} \) split:

...cont.
- there exists a diagonal from \( C_1 - C_2 \)
- if many, pick the one with endpoints \( P_1, P_2 \) closest to \( x_3 \) & \( x_4 \)
Finding a $\frac{n}{4} \cup \frac{3n}{4}$ split:

- ...cont.
- there exists a diagonal from $C_1 - C_2$
- if many, pick the one with endpoints $P_1, P_2$ closest to $x_3, x_4$
- Where is the apex of triangle $P_1P_2P_3$?
  \[ \text{Where isn't it?} \]
Finding a $\frac{n}{4} \times \frac{3n}{4}$ split:

...cont.

- there exists a diagonal from $C_1$ - $C_2$
- if many, pick the one with endpoints $P_1$ - $P_2$ closest to $x_3$ & $x_4$

- Where is the apex of triangle $P_1P_2P_3$?

→ Where is it not?

- $P_3$ is in $C_3$ or $C_4$

→ So we find a diag $C_1$ - $C_3$ or $C_2$ - $C_4$

Done
Finding a $\frac{n}{4}v \frac{3n}{4}$ split:

... cont.

- there exists a diagonal from $C_1 - C_2$
- if many, pick the one with endpoints $P_1P_2$ closest to $x_3$ & $x_4$

- Where is the apex of triangle $P_1P_2P_3$? ➔ where isn’t it?

- $P_3$ is in $C_3$ or $C_4$

➔ So we find a diag $C_1-C_3$ or $C_2-C_4$

$O(n)$ to find it ➔