More Convex Hull Algorithms

Incremental

$CH(n)$

$P_{n+1}$
More Convex Hull Algorithms

Incremental

\[ CH(n) \]

First test if \( \text{time?} \)

If yes? Then ignore
More Convex Hull Algorithms

Incremental

$\text{CH}(n)$

First test if $O(\log n)$

4. Yes? Then ignore

Update hull:
- Find 2 tangents
- Delete chain between

$P_{n+1}$
Binary search to find tangents

How do you delete the chain?
Incremental C.H.

Could also sort $x$ and add points in order

1. Suppose you have C.H.($p_1, \ldots, p_n$)
Incremental C.H.

Could also sort and add points in order

0) Suppose you have C.H.\((P_1 \ldots P_n)\)
1) Start with \(P_n \rightarrow P_{n+1}\)
**Incremental C.H.**

Could also sort \( x \) and add points in order

1. Suppose you have C.H.(p₁...pₙ)
2. Start with \( p_n \) \( p_{n+1} \)

\( \Rightarrow \) as with Graham scan
Incremental C.H.

Could also sort \( \rightarrow \) and add points in order

1) Suppose you have C.H. \((P_1 \ldots P_n)\)
2) Start with \(P_n\) \(\rightarrow \) \(P_{n+1}\)

\(\rightarrow\) as with Graham scan
Incremental C.H.

Could also sort and add points in order

1) Suppose you have C.H. (p_1...p_n)
2) Start with p_n, p_{n+1}
3) Pop vertices

\( \mathcal{O}(n) \) pops per increment
Incremental C.H.

Could also sort and add points in order

0) Suppose you have C.H. \((P_1...P_n)\)

1) Start with \(P_n\) \(\rightarrow P_{n+1}\)

2) Pop vertices \(\Rightarrow as \ with \ Graham \ scan\)

\(O(n)\) pops per increment

but also in total:

\(TIME = \text{Sort} + \Theta(n)\)
C.H. by DIVIDE & CONQUER

Goal: $O(n \log n) = T(n) = 2T\left(\frac{n}{2}\right) + \Theta(n)$

Heart of problem: how to merge two hulls in $O(n)$ time

UGLY better?
DIVIDE

$\varphi_1 \ldots \varphi_{\frac{n}{2}}$
$\varphi_{\frac{n}{2}} \ldots \varphi_n$

upper tangent

lower tangent (bridge)
FIND UPPER TANGENT/BRIDGE

ONLY upper hull points are candidates
find point-hull tangent
Alternate sides
L-find point-hull tangent

L-X_{max}

R-X_{min}
Alternate sides

Let find point-hull tangent
In each iteration, how do we find point-hull tangent?
In each iteration, how do we find point-hull tangent? $O(\log n)$
In each iteration, how do we find point-hull tangent?

\[ O(\log n) \]

But this could advance \( \Leftrightarrow \) discard only one point:

\[ T(n) = T\left(\frac{n}{2}\right) + n\log n \]

BAD
Alternate sides
4 find point-hull tangent

Just walk up.
"Linear" time per alternation

Total \(O(n)\)