More Convex Hull Algorithms

Incremental

First test if $n \leq \log n$

- Yes? Then ignore

Update hull:
- Find 2 tangents
- Delete chain between

$CH(n)$
Binary search to find tangents
Incremental C.H.

Could also sort $x$ and add points in order

$P_{n+1}$

0) Suppose you have C.H. $(P_1, ..., P_n)$
Incremental C.H.

Could also sort $\rightarrow$ and add points in order

0) Suppose you have $\text{C.H.}(p_1 \ldots p_n)$

1) Start with $p_n \rightarrow p_{n+1}$
**Incremental C.H.**

Could also sort and add points in order

1. Suppose you have C.H. \((p_1 \ldots p_n)\)
2. Start with \(p_n\)
3. Pop vertices

\(\Rightarrow\) as w/ Graham scan
Incremental C.H.

Could also sort \( \times \) and add points in order

0) Suppose you have C.H. \((p_1 \ldots p_n)\)

1) Start with \( p_n \) \( P_{n+1} \)

2) Pop vertices \( \Rightarrow \) as w/ Graham scan
Incremental C.H.

Could also sort $\rightarrow$ and add points in order

1) Suppose you have C.H.($p_1 \ldots p_n$)
2) Start with $p_n \rightarrow p_{n+1}$

$\Rightarrow$ as with Graham scan

$O(n)$ pops per increment

but also in total:

$\text{TIME} = \text{SORT} + O(n)$
C.H. by DIVIDE & CONQUER

**Goal:** \( O(n \log n) \) = \( T(n) = 2T(\frac{n}{2}) + O(n) \)

Heart of problem: how to merge two hulls in \( O(n) \) time

Ugly better?
Find upper tangent/bridge

Only upper hull points are candidates
Alternate sides
find point-hull tangent
Alternate sides
L find point-hull tangent
In each iteration, how do we find point-hull tangent?

\[ O(\log n) \]

binary search

But this could advance \(\Leftrightarrow\) discard only one point:

\[ T_n = T_{n/2} + n \log n \]

BAD
Alternate sides
4 find point-hull tangent

Just walk up.
"Linear" time per alternation

Total $O(n)$