$x, y : \text{match} \iff f(x) = f(y)$

**Claim:** $\exists \infty \text{ matches}$

trivial if $f = \text{const.}$ → Assume not.
\[ f(s) = f(t) = \frac{f(x) + f(y)}{2} \]

For any \( f(q) = (1-a)f(x) + af(y) \) \( \exists f(p) = f(q) \)

\( \exists \infty \) pairs w/ same value (pairwise)

Added restriction: I want a diametric match.

\( \langle \) pick one random pair. If \( \bullet \succ \bullet \) (H>L) then walk clockwise on both sides, keeping diameter.

Eventually from H>L we get H<L (at \( \delta = 180^\circ \))

So in between, there must be a position w/ H=L \( \square \)
Let \( f() \) be a continuous function on the surface of the sphere.
We know that $\exists$ 2 points on equator w/ equal temperature (or pressure, or...)

\[ \Rightarrow \text{ pick any equator or parallel, etc or any centrally symmetric closed curve} \]

\[ \begin{align*}
\{ \text{On loop}(a, b) \} & : \exists \text{ a diametric match.} \\
\text{true for any loop}(a, b) & : \infty \text{ number}
\end{align*} \]

\[ \Rightarrow \text{ facing us} \]

\[ \Rightarrow \text{ hidden on far side} \]

Note: $\bullet$ need not have same value
This set of matched points (*) cannot be maximal. Why?
Can find another loop$(a,b)$ : contradiction

There must be a closed curve separating $a,b$ s.t. every point on the curve is matched.

$\infty$ number of points on sphere w/ equal temperature to polar opposite!

(there may be others, not on the curve)
Now look at our curve and find any 2 polar opposites: \( a', b' \). We know \( f(a') = f(b') \).

Suppose we have a 2nd continuous function. Then *walk* while remaining opposite. Eventually we switch positions so somewhere on the curve \( f(a') = f(b') \), we also have \( g(a') = g(b') \).

\( \triangle 2 \) polar opposite points w/ equal temp + pressure
6 blue below
We can do this for any $K$ below, & wrap around $180^\circ$

Then just switch "below"$\rightarrow$"above" & complete $360^\circ$
Suppose we have 2 sets
Start with one halving line on blue.
Notice it has some wiggle room.
One side has more red points (+)

- Rotate (always ccw) and maintain blue split.
- Red points can enter and exit (+) ... in fact multiple times
  - However eventually (+) will be the complement of starting position.
  - but then (+) contains fewer red points:
    - we passed a position w/ red split
$R = 14$
$B = 8$

Ham sandwich cut
$R = 14$

$B = 8$

There is always a Ham sandwich cut passing through a red & blue point.
Not necessarily just by shifting any given cut.

But we can anchor & rotate on blue set, until we reach a red point.

Must happen because L & R will invert roles.
intersection of red & blue median levels

line through red & blue points that splits both point sets

not parallel
zones intersect