Minimizing distance to a set of \( n \) lines

- As mentioned, if we knew which cell we are in, its LP is \( O(n) \).

Thm: there always exists a pair of lines \( a, b \) s.t. \( \leq \frac{3}{4} \) of input lines intersect any quadrant.

This can be found in \( O(n) \) time.

In \( O(n) \) time we can find OPT on \( a \) & \( b \) & decide which quadrant contains global OPT. (median finding & gradient).

So we can assemble \( \frac{n}{4} \) lines into one constraint (new fake line): const. fraction removed: \( O(n) \) overall.

Duality & ham-sandwich.
Thm: there always exists a pair of lines $a,b$ s.t. $\leq \frac{3}{4}$ of input lines intersect any quadrant.

Partition: steep vs shallow slopes (by median slope)

Identify median level of each group

$\$ Intersect at $x$. 
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Partition: steep vs shallow slopes (by median slope)

Identify median level of each group

\( \Rightarrow \) Intersect at x.

Construct median overall slope, M.
Thm: there always exists a pair of lines $a, b$ s.t. $\leq \frac{3}{4}$ of input lines intersect any quadrant.

Partition: steep vs shallow slopes (by median slope)

Identify median level of each group

Intersect at $x$.

Claim: $M$ & $V$ partition the $n$ lines.

Construct median overall slope, $M$.

Then shift $M$ to $x$. 
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Partition: steep vs shallow slopes (by median slope)

Identify median level of each group $\&$ Intersect at $x$.

Construct median overall slope, $M$

Then shift $M$ to $x$.

Claim: $M$ & $V$ partition the $n$ lines

$\frac{1}{2}$ blue lines cross $V$ below $x$.

Also, shallower than $M$, so cross on left.

($\&$ v.v.: above $x$ $\&$ right on $M$.)
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Claim: $M$ & $V$ partition the $n$ lines.

Partition: steep vs shallow slopes (by median slope)

Identify median level of each group

$\Rightarrow$ Intersect at $x$.

Construct median overall slope, $M$

Then shift $M$ to $x$. 
In $\Theta(n^2)$ time we can construct all lines through pairs of points.

Use previous theorem to get Oja median in $O(n^2)$ time

(it works with weighted distances as well)

A more complicated $O(n\log^3 n)$ algorithm exists.