So if C.H. is \( \Omega(n \log n) \)

WHY is Jarvis March \( O(nh) \)?

\( n \cdot h < n \cdot \log n \)

\( O(nh) \) is just a more sensitive bound. & worst-case is \( O(n^2) \)

The knowledge of \( h \) is important.

Clearly if I told you \( h = 3 \) you wouldn't sort anything. ... \( \Omega(n) \)

We could try to get a lower bound in terms of \( n \) and \( h \)

\( \Rightarrow \) and also a better upper bound

latter
Quick Hull

- Maintain a convex polygon, for which
  - you know every vertex is on C.H.
  - all points inside are discarded forever

1) Find $X_{min}$ $X_{max}$ $Y_{min}$ $Y_{max}$
   Discard all points inside triangle/quad.
Quick Hull

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1) Find $X_{min}$ $X_{max}$ $Y_{min}$ $Y_{max}$
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Quick Hull

- Maintain a convex polygon, for which
  - you know every vertex is on C.H.
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1) Find $X_{\min}$ $X_{\max}$ $Y_{\min}$ $Y_{\max}$
   Discard all points inside triangle/quad.

2) Find extreme pts orthogonal to current polygon edges

3) Form new triangles, discard.

repeat
1) Find $X_{\text{min}}$, $X_{\text{max}}$, $Y_{\text{min}}$, $Y_{\text{max}}$. Discard all points inside triangle/quad.

2) Find extreme pts orthogonal to current polygon edges.

3) Discard, repeat.

- Every discard involves a quad, so $O(n)$.
- Number of regions to check doubles each time but ends up $O(n^2)$, so total = $O(n^2)$.
- Worst case: you never discard .... $O(n^3)$ total.

$O(n \log n)$ expected.
How would you find a diagonal?

- vertex to vertex
- inside polygon
- splits polygon into 2
• **Pick a convex vertex** $P_i$

• **Try** $P_{i-1} P_{i+1}$

• **If not intersected, done.**

$$P_{i-1} P_i P_{i+1} = \text{EAR}$$

What if the triangle is not empty?

Sweep parallel to $P_{i-1} P_{i+1}$

Join to first vertex found. Done

**Time:** $O(n)$
What if I prefer to start with a reflex vertex (assuming non-convex polygon)?

Can we still do it?

\[\text{Assume bisecting ray is vertical}\]
\[\text{w.l.o.g., by rotation}\]
\[\text{Let } v \text{ be the first point hit, on segment } xy\]
\[\text{If } v \text{ is a vertex: done}\]
Try \( \overline{Pi}x \). If not a diagonal then polygon must cross it

Then sweep inside \( \Delta xvp_i \) [rotational fixed at \( p_i \)]

Because \( \Delta \) not empty, will hit \( \overline{w} \).

QED ???? \( \text{No} \)

- What if \( \overline{wp_i} \) is a polygon edge?  
  \( \Rightarrow \overline{w} = \overline{Pi+1} \)

- What if \( x = \overline{Pi+1} \) ?
In both cases, $x$ must be below $p_i$. So $y$ must be above.
In both cases, $x$ must be below $p_i$. So $y$ must be above.

$O(n)$
TRIANGULATION

- partition of polygon into triangles

How do we know it exists for every n-gon?
TRIANGULATION

- partition of polygon into triangles

How do we know it exists for every n-gon?

- We can find one diagonal
- It splits the polygon into 2 smaller ones
- Use induction

\[ \Rightarrow \text{Assume every (n-1)gon can be triangulated} \]

Base case = \( \triangle \)

Worst case: diagonal splits n-gon \( \Rightarrow \) ear + (n-1)gon \( \Rightarrow O(n^2) \)
Meister's two-ear theorem

Every $n$-gon has $\geq 2$ non-overlapping ears. ($n > 3$)

$n = 4$

Notice: adjacent (convex) vertices can't form a pair.

proof?
proof

• Find a convex vertex.
  If it's an ear, cut it off.
  Apply induction.

  On sub-polygon, there are > 2 ears. They can't be at \( x \) \( \text{AND} \) \( y \).
  So one ear must not overlap ear.

Else: \( v \) is not an ear. Then find diagonal \( \overline{vs} \) as shown before.

\( \text{By induction, both sub-polygons have 2 ears} \)

As above, not both can be on \( \overline{vs} \).

On each side take the ear not at \( v \) \( \text{OR} \) \( s \).

\( \Box \)
Faster proof of 2-ear thin.

Every polygon has a triangulation.
Every triangulation has a dual tree.
Every tree has \( \geq 2 \) leaves.

- A leaf is an ear.