How long does it take to find one ear?

\[ \rightarrow \text{just finding diagonal and searching one side} : O(n^2) \]

\[ \rightarrow \text{A little better:} \]

\[ \text{still } O(n^2) \]

\[ \Rightarrow \text{List all convex vertices} \]

\[ \Rightarrow \text{Try each one} : O(n) \]

\[ \Rightarrow \text{by looking for reflex vertices inside} \]

\[ O(k) \]

\[ O(nk) \text{ if } k \text{ reflex} \]

If you have \( k \) convex, also \( O(nk) \)
How long does it take to find one ear?

\[ \Rightarrow \text{just finding diagonal and searching one side} = O(n^2) \]

You would have to make unbalanced cuts and go the wrong way each time to get \( O(n^2) \).

What about searching the smaller subpolygon?

Can get subpolygons with no original edges.
Every time you partition a GSP with a diagonal, one of the two sides is still a GSP (of the original).
**FINDING an EAR in LINEAR TIME**

- Pick any vertex: test if its an ear \( O(n) \) by sweep
  - If not \( \rightarrow \) form a diagonal \( O(n) \)
    - \( \rightarrow \) choose one side, arbitrarily
    - \( \rightarrow \) find ear on that side
      - \( \rightarrow \) find a particular ear (can't involve diagonal)

We have seen that when we cut a polygon with a diagonal, one of the 2 ears of each subpolygon is not on the diagonal.
our first choice of sub-polygon was arbitrary but we will choose the GSP side each time

So far this is still quadratic. Must choose diagonal carefully Ideas?
our first choice of sub-polygon was arbitrary but we will choose the GSP side each time

Form next diagonal from here How big is the next GSP?

Divide the problem size each time: prune & search
We have seen

Jarvis march $O(n \cdot h)$

Graham scan $O(n \cdot \log n)$

Quick hull $O(n^2)$ ... $O(n \cdot \log n)$ expected
Quickhull in a little more detail

- What is the worst case?

Form \[ \square \] zero deletions
Quickhull in a little more detail

- What is the worst case?

Search extremes
Find only one
Zero deletions
QuickHull in a little more detail

- What is the worst case?

Search extremes
Find only one
Zero deletions
QuickHull in a little more detail

- What is the worst case?

Search extremes
Find only one
Zero deletions
Etc
... $O(n^2)$
QuickHull in a little more detail

Also output-sensitive? $f(n, h)$

Notice that every new search happens when we find a convex hull vertex $O(n \cdot h)$
MORE CONVEX HULL ALGORITHMS

INCREMENTAL

First test if $O(\log n)$

- Yes? Then ignore

Update hull:
- Find 2 TANGENTS
- Delete chain between

$CH(n)$
\(CH(n)\)

Binary search to find tangents
Incremental C.H.

Could also sort $\rightarrow$ and add points in order

0) Suppose you have C.H.($P_1 \ldots P_n$)
Incremental C.H.

Could also sort \( \rightarrow \) and add points in order

0) Suppose you have C.H.(\( p_1 \ldots p_n \))
1) Start with \( p_n \rightarrow p_{n+1} \)
Incremental C.H.

Could also sort and add points in order

0) Suppose you have C.H. \((p_1 ... p_n)\)
1) Start with \(p_n\)
2) Pop vertices

\(\Rightarrow\) as w/ Graham scan
**Incremental C.H.**

Could also sort and add points in order:

1. Suppose you have C.H.\((P_1...P_n)\)
2. Start with \(P_n\) and \(P_{n+1}\)

\(\Rightarrow\) as w/ Graham scan
Incremental C.H.

Could also sort and add points in order

1) Suppose you have C.H. \( (p_1 \ldots p_n) \)
2) Start with \( p_n \rightarrow p_{n+1} \)

\[ \Rightarrow \text{as w/ Graham scan} \]

\( O(n) \) pops per increment but also in total:

\[ \text{TIME} = \text{SORT} + O(n) \]
C.H. by **DIVIDE & CONQUER**

**Goal**: $O(n \log n) = T(n) = 2T(n/2) + O(n)$

Heart of problem: how to merge two hulls in $O(n)$ time

[Diagram of two polygons merging into one]
DIVIDE

\[ \varphi_1 \cdots \varphi_{\frac{m}{2}} \quad \varphi_{\frac{m}{2}} \cdots \varphi_n \]

upper tangent

lower tangent (bridge)
Find upper tangent/bridge

Only upper hull points are candidates
find point-hull tangent
Alternate sides
4. Find point-hull tangent
Alternate sides

L - find point-hull tangent

R - X_{\min}

L - X_{\max}
In each iteration, how do we find point-hull tangent?

But this could **advance** ⇔ **discard** only one point:

$$T_n = T_{n/2} + n \log n$$

BAD
Alternate sides
4 find point-hull tangent

Just walk up.
"Linear" time per alternation

Total $O(n)$