Convex Hull of Simple Polygon

Ideas?
Remember Graham scan?

1) Sort
2) Locally establish convexity
Step 1 of Graham scan essentially constructs a polygon (what kind?)
Let's run part 2 again

The local checking can be visualized with 3 coins

1 2 3
Let's run part 2 again

The local checking can be visualized with 3 cows

Initial placement starting with two extreme points

→ must make a 3 turn
Let's run part 2 again.

The local checking can be visualized with 3 coins.

1 2 3

Advance while 5
Let's run part 2 again.

The local checking can be visualized with 3 cows.

Advanced until C

pop
Let's run part 2 again.

The local checking can be visualized with 3 cows.

1 2 3

popped & backtracked
Let's run part 2 again.

The local checking can be visualized with 3 coins:

1 2 3
Let's run part 2 again

The local checking can be visualized with 3 coins:

1 2 3
Let's run part 2 again

The local checking can be visualized with 3 coins

1 2 3
Let's run part 2 again

The local checking can be visualized with 3 coins

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1 2 3
Let's run part 2 again

The local checking can be visualized with 3 coins

This is the 3-coins algorithm a.k.a. Sklansky scan

Remember it's O(n) after polygonizing
Another example

Notice that polygon order ≠ angular order.
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etc
Once more
This method can fail
Notice that if we scanned clockwise, we'd have the opposite result.

What is "deep"?
$P_7 \ldots P_{14} : \text{Pocket}$

$P_7 P_{14} : \text{LID}$

$P_1 \ldots P_5 : \text{Pocket}$
Not externally visible, but weakly externally visible (WEV)

Every point in the pocket is visible from: < some point on \( \text{CID} \) \( \Rightarrow \) some point outside CH
CLAIM: Sklansky scan works on W.E.V.

- No convex hull vertex can be deleted: we only delete C
- Treat each pocket separately
  - can use induction
    - You are given a WEV pocket
    - Assume smaller WEV pockets can be handled
    - Discard a vertex to get a smaller WEV pocket
Claim: a pocket is WEV if its vertices are WEV.

For every edge, endpoints $s, t$, see $s', t'$ on lid.

$\Rightarrow$ 2 main cases $\Rightarrow$

In all cases, every interior point sees lid.
Base case: trivial

When we delete a right turn: we join $e$. $e$ doesn't intersect polygon. This would imply: one of the endpoints of $e$ would not be wev.

So we get a simple polygon but also every vertex is still WEP. No ray to infinity crossed through cut triangle.
MELKMAN'S ALGORITHM

• note: not the first O(n) C.H. algo, but it is the simplest.
• it is an online algorithm: at any time you have the C.H. of the current input. No pre-scanning.

• Start w/3 points
• Store them in a DEQUE:

\[
\begin{pmatrix}
3 \\
2 \\
1 \\
3
\end{pmatrix}
\]

• WLOG let 1 2 3 be \( \rightarrow \)
  \( \rightarrow \) then: bottom of deque: 2, 1, 3 = \( \leftarrow \)
  top of deque: 1, 2, 3 = \( \rightarrow \)

• Where could the next vertex be?
Inside region (yellow)

Do nothing

Also ignore the rest of the chain until some edge crosses \(1 \rightarrow 3\)

If this happens, we go to green or blue

(no difference from arriving in green/blue directly from \(3\))
Right region (green)

top of deque ok
bottom: last 3 not 8

re-established C.H. in both directions
Green is still the "right region"
Red is still "left region"
Always with respect to last point AND current hull.

1 should be deleted entirely
& 3 should still be deleted
Top of deque: 1, 2, 3, 4, not 5
"2" must be removed
3 must be discarded entirely.

\[
\begin{align*}
&\quad x \\
&\quad \{ \{ 1, 2, 3 \} \} \times \{ \{ 4 \} \} \\
&\quad \{ \{ 1, 2, 3 \} \} \\
&\quad \{ \{ 4 \} \} \times \{ \{ 1, 2, 3 \} \} \\
&\quad \{ \{ 4 \} \} \\
&\quad \{ \{ 1, 2, 3 \} \} \\
&\quad 4 \quad \underline{\text{X}}\quad 3 \\
&\quad \{ \{ 1, 2, 3 \} \} \\
&\quad \{ \{ 4 \} \} \\
&\quad \{ \{ 1, 2, 3 \} \} \\
&\quad \{ \{ 4 \} \} \\
&\quad \{ \{ 1, 2, 3 \} \} \\
&\quad 4 \\
\end{align*}
\]
Red: will pop 11 from top. could pop 9, 3, 2, not 1.

Green: will pop 11 from bottom. could pop 1, 2, not 3, 9

Blue: will pop 11 top & bottom. could pop 9 and/or 1.