Problem 1

For each of the following statements, indicate “True”, “False”, or “Not Known” (as in currently not known to anyone on planet earth).

1. P ⊆ NP
2. P ⊂ NP
3. If there is a polynomial time algorithm for 3SAT, then P = NP.
4. There exist problems in NP that are not NP-Complete.
5. If a language is in P, then so is its complement.
6. If a language is in NP, then so is its complement.
7. If L_1 ⊂ L_2 and L_1 is NP-Complete, then L_2 must also be NP-Complete.
8. If L_1 ⊂ L_2 and L_2 is NP-Complete, then L_1 must also be NP-Complete.
9. If L_1 ∈ NP and L_2 ∈ NP, then L_1 ∩ L_2 ∈ NP.
10. To show that a language L is NP-Complete, it suffices to show that (1) L ∈ NP and (2) If L ∈ P, then P = NP.

Problem 2

Barely Legal 3SAT is similar to NPSAT except that only one literal can be true in each and every clause.

BLSAT = \{〈φ〉 | φ is a satisfiable 3SAT formula where only one literal in each clause is true\}

Prove that 3SAT ≤_p BLSAT.

Hint: Each clause C_i in your original 3SAT formula should be replaced with 3 new clauses in your BLSAT formula. This conversion will require four new dummy variables.
Problem 3

SET-SPLITTING = \{ \langle S, C \rangle \mid S \text{ is a finite set and } C = \{C_1, \ldots, C_k\} \text{ is a collection of subsets of } S, \text{ for some } k > 0, \text{ such that elements of } S \text{ can be colored red or blue so that no } C_i \text{ has all its elements colored with the same color } \}

Prove that SET-SPLITTING is NP-Complete. (Hint: reduce SET-SPLITTING from 3SAT)