Problem 1

Each of the Turing machines that we have demonstrated so far has had a tape with a definitive start point on one end and an infinite number of squares on the other. Imagine then, a Turing machine that utilizes a tape that is infinite on both ends. Prove that these two machines are equivalent (i.e. they can perform the same computations).

Problem 2

Consider a tape that initially consists of a finite pattern of 1’s and 0’s, with the tape head positioned over the first square (see figure below). Write a Turing machine (formatting guidelines on the following page) that will extend this pattern infinitely along the tape.

Problem 3

Write a Turing machine that will subtract 1 from a binary number that is initially written on the tape. For convenience you may assume that the tape extends infinitely to the left, that the number to decrement is greater than 0, and that the first cell is marked with a ‘#’. The figure below depicts 1 being subtracted from 16 to arrive at 15.
Turing Machine Syntax

1 NAME, STATE, GAMMA, START, HALT are all one line each, are each followed by a colon (:), and must all be present.

2 States can be a sequence of printable ASCII characters that do NOT contain whitespace or quotes.
   • q0, q1, q3, q1# are all fine
   • q_Looking_for_a_one is legal, but frowned upon
   • “q state here” is illegal and will confuse the Turing Machine

3 Elements of Gamma can be a sequence of printable ASCII characters that are NOT whitespace or quotes. For clarity, do NOT reuse the names for states.
   • 0, 1, B, 1x, 0x, X, Y are all fine
   • “Crossed one out”, Ø, I are all illegal and will confuse the Turing Machine

4 The blank character MUST be B. You may assume that the tape is initially filled with blanks.

5 Recall delta is $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

6 Each line has one transition. Each part of the transition is separated by whitespace. L means head moves left. R means head moves right.

7 A transition to the halt state does not need to write to the tape or move the head. All other transitions do write to the tape AND move the head.

8 Any transitions not written in Delta will be treated as transitioning to the halt state.

9 Delta must end with END on a single line.

10 Anything after END will not be read by the simulator, so feel free to explain the machine and its states.

11 You may include comments in the code. A comment is on a line of its own starting with a semicolon.
NAME: Prof Monroe
STATE: q0 q1 qh
GAMMA: 0 1 B
START: q0
HALT: qh
DELTA:
q0 B q1 0 R
; This is a comment
q1 B q0 1 R
END

Comment however you like down here.
This TM computes the number .010101...