Problem 1

Of all the totally made up sports, I’d have to say that my favorite is the triple jump. In this spirit, consider a Turing machine that can only move right and left in sets of three. That is, in a single execution step, the tape head must move either three squares to the right or three squares to the left. As such, its transition function has the form:

\[ \delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{RRR, LLL\} \]

Is this Turing machine variant equivalent to the standard version (i.e. can it decide the same languages)? Prove why or why not.

Problem 2

Let \( x \) be a binary string of length \( n \), and let \( x_i, 0 \leq i \leq n - 1 \), be the character (bit) located at position \( i \). A string \( y \) is \( x \) bitwise doubled if for all \( y_i, y_i = x_{\lfloor i/2 \rfloor} \). Note that the empty string, bitwise doubled, is just the empty string.

For example, 010 bitwise doubled is 001100 and 00110 bitwise doubled is 0000111100.

Write a Turing machine (in mid-level pseudo code) to decide the following language:

\[ L = \{ y01x \mid x \in \{0, 1\}^* \text{ and } y \text{ is } x \text{ bitwise doubled } \} \]

** For a reference point on what we mean by “mid-level pseudo code”, please review the Recitation 2 solutions.

Problem 3

Prove that the following language, \( 3Z_{TM} \), is undecidable:

\[ 3Z_{TM} = \{ \langle M \rangle \mid M \text{ is a Turing machine that prints a number with at least 3 zeros} \} \]