Problem 1
Of all the totally made up sports, I’d have to say that my favorite is the triple jump. In this spirit, consider a Turing machine that can only move right and left in sets of three. That is, in a single execution step, the tape head must move either three squares to the right or three squares to the left. As such, its transition function has the form:

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{RRR, LLL\}$$

Is this Turing machine variant equivalent to the standard version (i.e. can it decide the same languages)? Prove why or why not.

Problem 2
Let $x$ be a binary string of length $n$, and let $x_i, 0 \leq i \leq n - 1$, be the character (bit) located at position $i$. A string $y$ is $x$ bitwise doubled if for all $y_i$, $y_i = x_{\lfloor i/2 \rfloor}$. Note that the empty string, bitwise doubled, is just the empty string.

For example, 010 bitwise doubled is 001100 and 00110 bitwise doubled is 0000111100.

Write a Turing machine (in mid-level pseudo code) to decide the following language:

$$L = \{ y01x \mid x \in \{0, 1\}^* \text{ and } y \text{ is } x \text{ bitwise doubled } \}$$

** For a reference point on what we mean by “mid-level pseudo code”, please review the Recitation 2 solutions.

Problem 3
Consider a variant of the universal Turing machine $3Z_{UTM}$ that takes a Turing machine description as input and determines whether or not it prints at least 3 zeros. Prove that $3Z_{UTM}$ is impossible to construct (i.e. the language $3Z_{UTM} = \{(M) \mid M \text{ is a Turing machine that prints a number with at least 3 zeros}\}$ is undecidable).