Announcement:

Your midterm is in class, on Thursday, October 12th.

Problem 1

A **Triple Jump Turing machine**, denoted $T_{i,j,k}$ is similar to an ordinary Turing machine, except that in addition to the ability to move left and right, this machine can jump to $i^{th}$, $j^{th}$, and $k^{th}$ positions of the tape as well. For example, the $M_{5,7,17}$ can move left one cell, move right one cell, jump to the $5^{th}$ cell on the tape, jump to the $7^{th}$ cell on the tape, or jump to the $17^{th}$ cell on the tape. Is this Turing machine variant equivalent to the standard Turing machine? Namely, can any language recognized by one be recognized by the other? Prove your answer.

Problem 2

Consider the problem

$$B = \{\langle M, w \rangle | M \text{ moves left at least 4 times during its computation on } w \}$$

Is B decidable? Why or why not?

Problem 3

Consider the problem

$$S = \{\langle M \rangle | M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w \}$$

Here $w^R$ is $w$ written in reverse. Show that $S$ is decidable.

Problem 4

Consider the problem

$$T = \{\langle M \rangle | M \text{ is a Turing machine that accepts } w^R \text{ whenever it accepts } w \}$$

Here $w^R$ is $w$ written in reverse. Show that $T$ is undecidable.

Problem 5

Show there exists a language $L$ such that neither $L$ nor $\overline{L}$ is Turing recognizable.