Problem 1
Prove that the following language is undecidable:

\[ L_1 = \{ \langle M_1, M_2, M_3 \rangle \mid L(M_1) \cup L(M_2) = L(M_3) \} \]

Problem 2
Prove that the following language is undecidable:

\[ L_2 = \{ \langle M \rangle \mid M \text{ does not accept a string beginning with } 1 \} \]

Problem 3
Prove that \( \overline{A_{TM}} \leq_m L_2 \).
Please complete one of the following problems for extra credit. Submit your solution along with your solutions to HW5.

Problem 1
Consider the problem of determining whether a Turing machine $M$ on an input $w$ ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

Problem 2
Consider the problem of determining whether a Turing machine $M$ on an input $w$ ever attempts to move its head left at any point during its computation. Formulate this problem as a language and show that it is decidable.

Problem 3
Suppose $A$ and $B$ are languages and $A \leq_m B$

Prove that if $A$ is undecidable, then $B$ is undecidable.