Problem 1
Consider the following language:

DOUBLE-CLIQUE = \{ (G, k) \mid G \text{ has at least 2 cliques, each of size greater than or equal to } k \}

Prove that DOUBLE-CLIQUE is NP-Complete.

Problem 2
For any $k$, the language $k$-COLOR is defined to be the set of (undirected) graphs whose vertices can be colored with at most $k$ distinct colors, in such a way that no two adjacent vertices are colored the same color. Prove that 4-COLOR is NP-Complete.

Problem 3
Nobody’s Perfect 3SAT is the problem where given a boolean formula in conjunctive normal form with 3 literals in each clause you want to know if there is a satisfying assignment where at least one literal in each and every clause evaluates to false.

NPSAT = \{ (\phi) \mid \phi \text{ is a satisfiable 3SAT formula where no clause evaluates to all true} \}

Prove that NPSAT is NP-complete.

(Note: A boolean formula consists only of variables. You can’t use the constants TRUE or FALSE.)