THE PUMPING LEMMA
A non-regular language?
An FSA that decides...

$L = \{ \langle F, w \rangle \mid F \text{ is an FSA that accepts } w \}$
An FSA that decides...

\[ L = \{ w \mid w \text{ valid URL} \} \]
An FSA that decides...

$L = \{ \ w \mid w \text{ is a section of comments} \ \}$

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An FSA that decides...

$L = \{ w \mid w \text{ is a word in the dictionary} \}$
An FSA that decides...

\[ L = \{ w \mid w \text{ is of the form } 0^31^3 \} \]
An FSA that decides...

\[ L = \{ w \mid w \text{ is of the form } 0^n1^n \} \]
L = \{ w \mid w \text{ is of the form } 0^n1^n \} \\

An FSA with p states
\[ L = \{ w \mid w \text{ is of the form } 0^p1^p \} \]

An FSA with \( p \) states
Input string of size \( 2p \)
$L = \{ w \mid w \text{ is of the form } 0^p 1^p \}$
\[ L = \{ w \mid w \text{ is of the form } 0^p 1^p \} \]
\[ L = \{ w \mid w \text{ is of the form } 0^p1^p \} \]
The Pumping Lemma

If $L$ is a regular language then there is a number $p$, such that any input string longer than $p$ can be split into sections $x$, $y$, and $z$, such that:

1.) For each $i \geq 0$, $xy^iz \in L$
2.) $|y| > 0$
3.) $|xy| \leq p$
The Pumping Lemma

If L is a regular language then there is a number p, such that any input string longer than p can be split into sections x, y, and z, such that:

1.) For each $i \geq 0$, $xy^iz \in L$

2.) $|y| > 0$

3.) $|xy| \leq p$
\[ L = \{ \, ww \mid w \in \{0, 1\}^* \, \} \]

1.) Assume \( L \) is regular and thus has pumping length \( p \)

2.) Let the input be \( 0^p10^p1 \) (\( w = 0^p1 \) )

3.) \( 0^p10^p1 = xyz \), so where do we put \( y \)?
The Pumping Lemma

If \( L \) is a regular language then there is a number \( p \), such that any input string longer than \( p \) can be split into sections \( x, y, \) and \( z \), such that:

1.) For each \( i \geq 0, \ xy^i z \in L \)

2.) \( |y| > 0 \)

3.) \( |xy| \leq p \)
L = \{ w w \mid w \in \{0, 1\}^* \}

1.) Assume L is regular and thus has pumping length p

2.) Let the input be $0^p10^p1$ ($w = 0^p1$)

3.) $0^p10^p1 = xyz$, so where do we put y?
An FSA that decides...

$L = \{ 0^i1^j | i > j \}$
L = \{ 0^i 1^j \mid i > j \}

1.) Assume L is regular and thus has pumping length p

2.) Let the input be $0^{p+1}1^p$

3.) $0^{p+1}1^p = xyz$, so where do we put y?
The Pumping Lemma

If L is a regular language then there is a number p, such that any input string longer than p can be split into sections x, y, and z, such that:

1.) For each $i \geq 0$, $xy^iz \in L$
2.) $|y| > 0$
3.) $|xy| \leq p$
$$L = \{ 0^i1^j \mid i > j \}$$

1.) Assume $L$ is regular and thus has pumping length $p$

2.) Let the input be $0^{p+1}1^p$

3.) $0^{p+1}1^p = xyz$, so where do we put $y$?
The Pumping Lemma

If $L$ is a regular language then there is a number $p$, such that any input string longer than $p$ can be split into sections $x$, $y$, and $z$, such that:

1.) For each $i \geq 0$, $x y^i z \in L$

2.) $|y| > 0$

3.) $|xy| \leq p$
\[ L = \{ 0^i1^j \mid i > j \} \]

1.) Assume \( L \) is regular and thus has pumping length \( p \)

2.) Let the input be \( 0^{p+1}1^p \)

3.) \( 0^{p+1}1^p = xyz \), so where do we put \( y \)?
\[ L = \{ 0^i1^j \mid i > j \} \]

1.) Assume \( L \) is regular and thus has pumping length \( p \)

2.) Let the input be \( 0^{p+1}1^p \)

3.) \( 0^{p+1}1^p = xz \), pump \( y \) 0 times?
An FSA that decides...

$L = \{ w \mid w \text{ is of the form } 0^31^3 \}$
An FSA that decides...

$L = \{ w \mid w \text{ is of the form } 0^n1^n \}$