Problem 1
Prove that the following language is regular by constructing either a DFA or an NFA to recognize it. Then prove the same language is regular by constructing a regular expression for it.

\[ L = \{ x \in \{a,b,c\}^* \mid \text{the last 3 characters of } x \text{ are not } a's \} \]

Regular expression: \((a \cup b \cup c)^* \cup (b \cup c)^3 \cup (b \cup c)^2 \cup (b \cup c) \cup \epsilon\)

Problem 2
Show by giving an example that, if \( M \) is an NFA that recognizes language \( C \), swapping the accept and non-accept states in \( M \) does not necessarily yield a new NFA that recognizes the complement of \( C \). Is the class of languages recognized by NFAs closed under complement? Why or why not?

Let NFA \( A \) be

over the alphabet \( \{0,1\} \). \( L(A) = S^* \), so \( A \) accepts '1'.

Let \( B \) be the NFA obtained from \( A \) by switching the accept and reject states:
Since $B$ also accepts all strings of $S^*$, $L(B) = S^*$ and $B$ accepts ‘1’. Therefore, $L(B) \neq \overline{L(A)}$.

The class of languages recognized by NFA’s is closed under complement.

**Proof.** Let $C$ be an NFA. Since we can convert $C$ into a DFA, there is a DFA $M$ that recognizes $L(C)$. So, $L(C)$ is regular. By the proof in Problem 2, there is a DFA $N$ that recognizes $\overline{L(C)}$. Since every DFA is an NFA, there is an NFA that recognizes $\overline{L(C)}$. Therefore, the class of languages recognized by NFA’s is closed under complement. \qed