COMP170 Fall 2017, Recitation 7

For each of the following sets, give the smallest complexity class in which it is contained. Classes are listed from smallest to largest. Write your choice A through D, and then try to justify your answer. Your choices are:

- A. Decidable
- B. Undecidable but Recognizable
- C. Undecidable and Unrecognizable but its complement is Recognizable
- D. Undecidable and both it and its complement are Unrecognizable

i. \( \{ (M, x, y) \mid M \text{ loops on both } x \text{ and } y \} \)
   Solution: Group C. Reduction from HALT complement.

ii. \( \{ (M_1, M_2) \mid M_2 \text{ accepts every input that } M_1 \text{ accepts } \} \)
    Solution: Group D. Alpha-equivalent to EQTM

iii. \( \{ (M_1, M_2) \mid M_1 \text{ only accepts strings beginning with 0 and } M_2 \text{ only accepts string beginning with 1 } \} \)
    Solution: Group D. Can’t check that either language only does anything, would have to run forever. Complement reduction from ATM complement.

iv. \( \{ (M, w) \mid M \text{ does not accept } w \text{ within } |w|^2 \text{ time steps.} \} \)
    Solution: Group A. Operates in finite time.

v. \( \{ (M) \mid M \text{ enters state } q_7 \text{ on input 7 } \} \)
    Solution: Group B. Reduction from ATM

vi. \( \{ (M_1, M_2) \mid M_1 \text{ loops on every input rejected by } M_2 \} \)
    Solution: Group C. Complement is \( M_1 \) does not loop on every input rejected by \( M_2 \), which means we need only find one counterexample, which is recognizable.