Show that the following language $L$ is in NP.

$L = \{ (S,k) \mid S \text{ is a finite set of binary strings, } k \text{ is a positive integer,}
\text{ and there is some binary string } w \text{ s.t. } |w| < k \text{ and } \forall x \in S, x \text{ is a substring of } w \}$

Also, show that if $|S| = 2$, then we can decide membership of $L$ in polynomial time.