Prove that
\( E_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is } \emptyset \} \)
is undecidable.

Proof by reduction: \( A_{TM} \) to \( E_{TM} \)

Initial Assumption: There exists a TM, \( D_E(M) \), that can decide \( E_{TM} \).

We will now build a decider for \( A_{TM} \) that takes input \( \langle M, w \rangle \) and uses \( D_E \) to determine if \( M \) accepts \( w \). This decider, \( D_A \), will function as follows:

1.) Using input \( \langle M, w \rangle \), construct a new machine \( M' \) that works as follows:
   - \( M' \) on input \( x \):
     - If \( x \neq w \), REJECT
     - If \( x = w \), simulate \( M \) on \( w \) and DWID (do what it does)

2.) Run \( D_E \) on input \( \langle M' \rangle \)
   - If \( D_E \) accepts \( M' \), REJECT
   - If \( D_E \) rejects \( M' \), ACCEPT

Case 1: \( \langle M, w \rangle \in A_{TM} \) In this case \( M \) accepts \( w \) and thus \( M' \) will accept when its input is equal to \( w \). Thus \( L(M') \) is not empty, so \( D_E \) rejects \( \langle M' \rangle \) causing \( D_A \) to accept \( \langle M, w \rangle \).

Case 2: \( \langle M, w \rangle \notin A_{TM} \) This means that \( M' \) will reject all possible inputs and \( L(M') = \emptyset \). \( D_E \) will thus accept \( \langle M' \rangle \) causing \( D_A \) to reject \( \langle M, w \rangle \).