HW: due Thursday, February 26th

Note: No class on Thursday, February 19th: Tufts is on a Monday schedule!

The *fault-tolerant* version of the k-center problem with triangle inequality has an additional input $\alpha \leq k$ which specifies the number of centers that each vertex must be connected to. In other words, we assume that up to $\alpha - 1$ centers might be closed, and so the fault-tolerant cost for a vertex is its distance to its αth closest center. The problem is to pick k centers so that the maximum fault-tolerant cost of a vertex is minimized.

A set $S \subseteq V$ in an undirected graph $H = (V, E)$ is an *α-dominating set* if each vertex $v \in V$ is adjacent to at least α vertices in S (we consider a vertex to be adjacent to itself). Let $dom_\alpha(H)$ denote the size of a minimum cardinality α-dominating set in H.

1. Let I be an independent set in H^2. Show that $\alpha|I| \leq dom_\alpha(H)$.
2. Give a factor 3 approximation algorithm for the fault-tolerant k-center problem (Hint: Compute a maximal independent set M_i in G_i^2, for $1 \leq i \leq m$. Find the smallest index i such that $|M_i| \leq \lfloor \frac{k}{\alpha} \rfloor$, and moreover, the degree of each vertex of M_i in G_i is $\geq \alpha - 1$.)