HW 2: due Tuesday, February 20

1. A certain commodity is produced at two factories $x_1$ and $x_2$. The commodity is to be shipped to markets $y_1$, $y_2$ and $y_3$ through the network shown below. What is the maximum amount that can be shipped from the factories to the markets?

2. A vertex cover of a graph is a set of vertices $C$, such that every edge has at least one endpoint in $C$. Use network flows to prove the Kőnig-Egerváry theorem, i.e. if $G$ is bipartite, then the size of the maximal matching is equal to the size of the minimum vertex cover.

3. The fault-tolerant version of the $k$-center problem with triangle inequality has an additional input $\alpha \leq k$ which specifies the number of centers that each vertex must be connected to. In other words, we assume that up to $\alpha - 1$ centers might be closed, and so the fault-tolerant cost for a vertex is its distance to its $\alpha$th closest center. The problem is to pick $k$ centers so that the maximum fault-tolerant cost of a vertex is minimized. A set $S \subseteq V$ in an undirected graph $H = (V,E)$ is an $\alpha$-dominating set if each vertex $v \in V$ is adjacent to at least $\alpha$ vertices in
S (we consider a vertex to be adjacent to itself). Let $\text{dom}_\alpha(H)$ denote the size of a minimum cardinality $\alpha$-dominating set in $H$.

(a) Let $I$ be an independent set in $H^2$. Show that $\alpha|I| \leq \text{dom}_\alpha(H)$.

(b) Give a factor 3 approximation algorithm for the fault-tolerant $k$-center problem (Hint: Compute a maximal independent set $M_i$ in $G_i^2$, for $1 \leq i \leq m$. Find the smallest index $i$ such that $|M_i| \leq \left\lfloor \frac{k}{\alpha} \right\rfloor$, and moreover, the degree of each vertex of $M_i$ in $G_i$ is $\geq \alpha - 1$.)