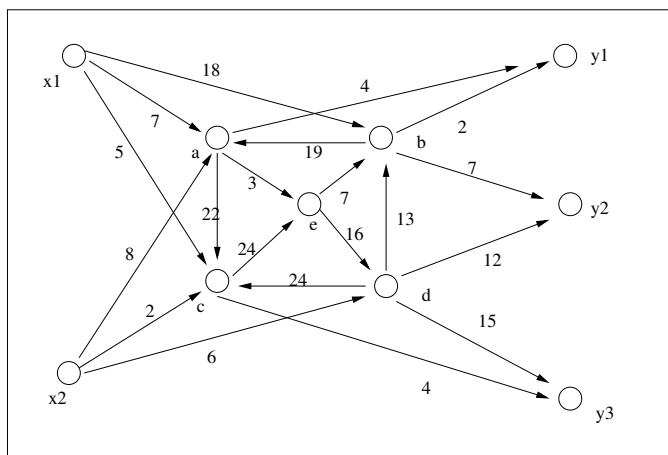


## HW 2: Due Tuesday, Feb 28 in class

1. A certain commodity is produced at two factories  $x_1$  and  $x_2$ . The commodity is to be shipped to markets  $y_1$ ,  $y_2$  and  $y_3$  through the network shown below. What is the maximum amount that can be shipped from the factories to the markets?



2. Let  $G$  be a directed graph with source  $s$  and sink  $t$ . Suppose the capacities are specified not on the edges of  $G$  but on the vertices (other than  $s, t$ ); for each vertex there is a fixed limit on the total flow through it. There is no restriction on flow through the edges. Show how to use the ordinary network flow theory to determine the maximum capacity of a feasible flow from  $s$  to  $t$  in the vertex-capacitated graph  $G$ .
3. A *vertex cover* of a graph is a set of vertices  $C$ , such that every edge has at least one endpoint in  $C$ . Use network flows to prove the Kőnig-Egeváry theorem, i.e. if  $G$  is bipartite, then the size of the maximal matching is equal to the size of the minimum vertex cover.