HW 3: due Tuesday, April 11

1. Prove Chevychev's inequality using Markov's inequality

2. The one-dimensional bin packing problem is defined as follows: there are n objects of size $a_1, \ldots, a_n$, where for each i, $0 < a_i < 1$. These items must be packed into bins of size 1, and we wish to use as few bins as possible. Give a 2-approximation for this problem.

3. Fix a parameter ϵ with $0 < \epsilon < 1$ Given an unweighted graph, with vertices labeled from $v_1 \ldots v_n$. For each vertex, define its *local neighborhood* to be its closest n^ϵ neighbors (breaking ties lexicographically by vertex name). Prove that for any graph G we can find, in polynomial time, a sparse subset of the vertices L (which we call the landmarks) such that: 1) For all vertices, there exists at least one landmark in its local neighborhood and 2)$|L| = O(n^{1-\epsilon} \log n)$.