HW 4: due Tuesday, April 24

1. Consider the problem of scheduling $n$ identically-sized tasks on 2 machines, with a set of pre-requisites given by a partial order $<$. Give a polynomial time algorithm that finds the optimal length schedule.

2. Consider a bipartite graph $G = (U, V, E)$ on $2n$ vertices that contains a perfect matching. Suppose the vertices in $U$ arrive in an online fashion and the edges incident to each vertex $u \in U$ are revealed when $u$ arrives. When this happens, the algorithm may match $u$ to a previously unmatched adjacent vertex in $V$, if there is one. Such a decision, once made is irrevocable. The objective is to maximize the size of the resulting matching.

   (a) Consider the algorithm that always matches a vertex in $U$ if a match is possible. Show that this algorithm achieves a competitive ratio of $1/2$.

   (b) Consider the following randomized online matching algorithm: 1) Randomly rank all the vertices in $V$. 2) As each vertex in $U$ arrives, match it with the highest rank vertex remaining to which it has an edge. Show that this algorithm does better in expectation than the deterministic algorithm above: you might want to read: Birnbaum, B. and Mathieu, C., 2008. On-line bipartite matching made simple. ACM SIGACT News, 39(1), pp.80-87.