Class exercise: Coding with scaled integers

COMP 40
October 19, 2009

Group

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<th>Keeper of the record:</th>
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<td>Other group members:</td>
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Coding coefficients of the Discrete Cosine Transform

Your homework requires you to convert luminance of a $2 \times 2$ block of pixels using the Discrete Cosine Transform, which produces coefficients $a$, $b$, $c$, and $d$. In principle, $b$, $c$, and $d$ range over the interval $[-0.5, +0.5]$, but in practice almost all values fall into the range $[-0.3, +0.3]$. You must exploit this property or your compressed images will have ugly artifacts.

Your job is to code the real numbers $b$, $c$, and $d$ as signed, scaled, 5-bit integers. Let us assume that a signed 5-bit integer can represent exactly the set of integers $\{n | -15 \leq n \leq +15\}$. (This assumption is almost but not quite the truth.) Please write C functions encode and decode with these prototypes:

```c
int encode(float x);
float decode(int n);
```

Your functions should satisfy four algebraic laws and an $\forall - \exists$ property:

- $\text{encode}(\text{decode}(n)) = n$ when $|n| \leq 15$
- $\text{decode}(\text{encode}(x)) \approx x$ when $-0.3 \leq x \leq 0.3$
- $\text{encode}(x) = \text{encode}(+0.3)$ when $x > +0.3$
- $\text{encode}(x) = \text{encode}(-0.3)$ when $x < -0.3$
- $\forall n : |n| \leq 15 : \exists x : |x| \leq 0.3 : \text{encode}(x) = n$ (all bits are used for the interval $[-0.3, +0.3]$)

where $x \approx y$ is defined to mean $|x - y| \leq \frac{1}{30}$.

1. Write the code.
2. Explain how you would test it.

Please return your work to the course staff.