

## HW 1: due Thursday, September 13 in class

1. Let  $G$  be a 3-regular graph with 18 edges and  $n$  vertices. Find  $n$ .
2. Let  $G$  be a simple graph with 50 edges. What is the least number of vertices  $G$  may have?
3. For a graph  $G$  where  $|G| \geq 2$ , prove that at least two vertices of  $G$  have the same degree.
4. Prove that an  $n$ -vertex graph cannot be bipartite if it has more than  $\frac{1}{4}n^2$  edges.
5. Show that in any group of 6 people, there must be either 1) a group of three mutual friends (i.e. a “triangle” of friends) or 2) a group of three mutual non-friends. (Where “friend” is defined as a symmetric relation among pairs of people, i.e. whenever Alice considers Beth a friend, it’s assumed that Beth also considers Alice a friend).