

HW 2: due Thursday, September 20 in class

Note: for the rest of this class, assume all graphs are SIMPLE unless otherwise stated.

1. Show that in any group of two or more people, there are always two with exactly the same number of friends in the group.
2. Show that if G is a tree with $\Delta \geq k$, then G has at least k vertices of degree 1. (Recall Δ is the maximum degree of G).
3. A *center* of G is a vertex u such that $\max_{v \in V} d(u, v)$ is as small as possible. Show that a tree has either exactly one center or two adjacent centers.
4. Let G be connected and let $e \in E$. Show that e is in every spanning tree of G if and only if e is a cut edge of G .
5. Prove that the graph obtained from K_n by deleting an edge has $(n - 2)n^{n-3}$ spanning trees. (Recall that K_n is the complete graph on n vertices).