HW 4: due Thursday, October 4 in class

1. For which values of r is $K_{r,r}$ Hamiltonian?

2. A mouse eats its way through a 3x3x3 cube of cheese by eating all the 1x1x1 subcubes. If it starts at a corner subcube and always moves on to an adjacent subcube (where adjacent means sharing a face of area 1), can it do this and eat the center subcube last? Give a method or prove impossible (Ignore gravity).

3. The *line graph* of a graph $L(G)$ is formed as follows; there is a vertex $v \in L(G)$ for every *edge* in G, and two vertices are adjacent in $L(G)$ iff their corresponding edges share an endpoint in G.

   (a) Find a 2-connected non-Eulerian graph whose line graph is Hamiltonian.

   (b) Prove that $L(G)$ is Hamiltonian if and only if G has a closed trail that connects at least one endpoint of at least one edge.

4. The *cartesian product* graph of G and H, is the graph with $|G|x|H|$ vertices, one for every ordered pair (g, h) of vertices with $g \in G$ and $h \in H$, and edge set defined as follows: (g, h) is adjacent to (g', h') if either $g = g'$ and hh' is an edge in H or $h = h'$ and gg' is an edge in G.

   (a) Draw the graph that's the Cartesian product of G = a path on 4 vertices and H = a path on 5 vertices.

   (b) Draw the Cartesian product of a 3-cycle and a 4-cycle.

   (c) Prove that the Cartesian product of two Hamiltonian graphs is Hamiltonian.

   (d) As in HW 3, we define the *k-dimensional hypercube* Q_k to be the following graph. There is a vertex for each binary string of length k. Two vertices are connected by an edge if and only if they differ in exactly one bit position. Prove that Q_k is Hamiltonian.

5. Let G be a simple graph that is not a forest and the length of the shortest cycle is at least 5. Prove that $\overline{G}$ is Hamiltonian. (Hint: use Ore's condition).