

HW 7: due Thursday, November 1 in class

1. A plane graph is called *self-dual* if it is isomorphic to its dual.
 - (a) Show that if an n -vertex graph G is self dual, then G has $2n - 2$ edges.
 - (b) For each $n \geq 4$, construct a simple n -vertex self-dual plane graph.
2. Let G be a plane graph. Show that G^{**} is isomorphic to G if and only if G is connected.
3. Show that if G is a connected planar graph with n vertices and girth $k \geq 3$, then G has at most $k(n - 2)/(k - 2)$ edges.
4. Show that if G is a simple planar graph with ≥ 11 vertices, then the complement of G is non-planar.
5. An *outerplanar graph* is a graph that can be embedded in the plane so that every vertex appears on the outside face. Use the 4-color-theorem to prove that any outerplanar graph is 3-colorable.
6. (Extra credit) Prove that every outerplanar graph is 3-colorable *without* using the 4-color theorem.