HW 1: due Tues, September 13th

1. Find necessary and sufficient conditions for the numbers s and t to make the LP problem:

$$\text{Maximize } x_1 + x_2$$

$$\text{subject to } sx_1 + tx_2 \leq 1$$

$$x_1, x_2 \geq 0$$

   (a) have an optimal solution
   (b) be infeasible
   (c) be unbounded

2. A meat packing plant produces 480 hams, 400 pork bellies and 230 picnic hams every day; each of these products can be sold either fresh or smoked. The total number of hams, bellies and picnics that can be smoked during a normal working day is 420, in addition up to 250 products can be smoked on overtime at a higher cost. The *net* profits are as follows:

	Fresh	Smoked (reg time)	Smoked (overtime)
Hams	$8	$14	$11
Bellies	$4	$12	$7
Picnics	$4	$13	$9

The objective is to find the schedule that maximizes the total net profit. Formulate as an LP in standard form.

3. Prove that every tree has at most 1 perfect matching.

4. Let B be a k-regular bipartite graph (i.e. all vertices have the same degree). Prove that B must have a perfect matching.

5. Let $G = (X, Y)$ be a bipartite simple graph with $|X| = |Y| = n$, and $X = x_1 \ldots x_n$, $Y = y_1, \ldots y_n$. Define the matrix $A = (a_{ij})$ by

$$a_{ij} = \begin{cases} 1, & \text{if } x_i y_j \in E(G) \\ 0, & \text{otherwise} \end{cases}$$

Please show the following.

(a) Give an example of a bipartite graph G for which the determinant of A is not 0.

(b) Prove that if G has no perfect matching, then det A is 0.

(c) Show that the converse is false, i.e. there are graphs with perfect matchings, for which the determinant of A is 0.

(d) Is there a polynomial time algorithm that determines if a bipartite graph has an odd number of perfect matchings?