

HW 4: due Tues, October 17th

1. A physicist takes measurements of a variable $y(x)$, the results are in the form of pairs (x_i, y_i) . The physicist wishes to find the straight line that fits this data best, in the sense that the maximum vertical distance between any point (x_i, y_i) and the line is as small as possible. Formulate this problem as an LP. Why might you decide to solve the dual?
2. Solve the following problem by the simplex method:

$$\text{Maximize } 3x_1 + 2x_2 + 4x_3$$

subject to

$$x_1 + x_2 + 2x_3 \leq 4$$

$$2x_1 + 3x_3 \leq 5$$

$$2x_1 + x_2 + 3x_3 \leq 7$$

$$x_1, x_2, x_3 \geq 0$$

3. Answer yes or no and prove your answer: can a pivot of the simplex algorithm move the feasible point a positive distance in R^n while leaving the cost unchanged?
4. Can a vector which has just left the basis in the simplex algorithm re-enter on the very next pivot?

5. The maximum flow problem on the directed graph $G = (V, E)$ with capacity function u (and lower bounds 0) can be formulated by the following linear program:

maximize w

subject to

$$\sum_j x_{ij} - \sum_j x_{ji} = w \text{ if } i = s ; 0, \text{ if } i \neq s, t; -w \text{ if } i = t$$

$$x_{ij} \leq u_{ij}$$

$$0 \leq x_{ij}$$

- (a) Show that its dual is equivalent to:

$$\min \sum_{(i,j) \in E} u_{ij} y_{ij}$$

subject to

$$z_i - z_j + y_{ij} \geq 0 \quad (i, j) \in E$$

$$z_s = 0, z_t = 1$$

$$y_{ij} \geq 0$$

- (b) (extra credit-hard) Given any (not necessarily integral) optimal solution y^*, z^* of value W^* to this dual linear program, show how to construct from z^* a cut separating s from t of value W^* .

Hint: make a clever use of complementary slackness

- (c) Deduce the max-flow–min-cut theorem: the value of the maximum flow from s to t is equal to the value of the minimum cut separating s and t .