HW 6: due Tues, December 6th

Suppose in our city we have n destinations connected by roads whose distances are represented by weighted edges where weights satisfy the triangle inequality (i.e. $d(u, v) \leq d(u, w) + d(w, v)$, for all possible destinations u, v, and w. (You can assume the city is connected).

Define a destination's "local neighborhood" to be the $\sqrt{n}$ nodes that are closest to the destination, including the destination itself and breaking ties arbitrarily.

1. A landmark is just a chosen destination. Show that you can always choose a set of $O(\sqrt{n} \log n)$ *landmarks* so that every destination has at least one landmark in its local neighborhood.

2. Consider the following route from destination u to destination v. If u is in v's local neighborhood, we route directly along the shortest path. Otherwise, instead of routing from u to v, we route from u to v's closest landmark, and then from v's closest landmark to v. Show that the length of this route is never more than 3 times the distance of the optimal route.

3. A *k-spanner* of a network is a subset of the edges of the original network such that shortest distances in the spanner are at most k times shortest distances in the original network. Show that any (weighted) graph has a 3-spanner with at most $O(n^{3/2} \log n)$ edges.