DistanceMaps
representing smooth surfaces in sampled volumes

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Why Represent Surfaces?
volumes are best for:

- representing amorphous substances
  - smoke, fire, clouds

- exploring sampled data
  - medical, geophysical, computational fluid dynamics

- representing object interiors
  - visualization, haptics
  - object deformation
  - cutting, carving

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Why Represent Surfaces?

representing interior structure:
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applications:
– simulation
– animation
– gaming

requires high quality rendering and shading
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representing interior structure:

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requires high quality rendering and shading

→ need to represent object surfaces

Why is it Hard?

- sampled data has a limited resolution
- volume samples do not necessarily lie on object surfaces
- the positions and normals of surface points must be interpolated from the sampled data ... but ... interpolation is expensive and has limited accuracy
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**Image Shading**

- in rendering, surfaces are highlighted using shading
- shading uses the *surface normal* to assign voxel colors based light sources, object color, and viewpoint
- the *surface normal* must be estimated from the sampled volume data
shading is performed on pixels in the rendered image using:

- **z-buffer depths**

- **gradients of the z-buffer depths**
  (Horn 1982, Gordon and Reynolds 1985, Bright and Laflin 1986)

**hybrid volumes** incorporate information about the surface into the volume data:

- store surface normals with each voxel

- store identification of the surface passing through each surface voxel
  (Yagel et al 1992)
surface normals are estimated as the local gradient of the gray-scale data:

- gradients are calculated using a gradient estimator such as:
  \[ g(x, y, z) = (i(x+1, y, z) - i(x-1, y, z), \\ i(x, y+1, z) - i(x, y-1, z), \\ i(x, y, z+1) - i(x, y, z-1)) \]

(Hohne et al 1990, Levoy 1988, Crawford et al 1988)

in fact, there are many types of interpolation and gradient estimation filters:

- e.g. linear interpolation, cubic interpolation
- e.g. central differences, spline-based derivative filters, cubic derivative filter
- analysis of filter design:
Why is it a Hard Problem

– in most volume data, image intensities change abruptly at surfaces
– abrupt intensity changes correspond to high spatial frequencies
– high spatial frequencies require high sampling rates (Nyquist)

→ inadequate sampling rates result in aliasing that is exaggerated by derivative filters
Surface Normal Estimation

- a binary object (white inside, black outside)
- the sampled binary object
- the central differences gradient estimator is a poor estimator of the edge's normal

Surface Normal Estimation

- aliasing due to inadequate sampling of the volume data
Intensity-based Volumes

– for alias-free reconstruction, signals must be sampled at twice the highest frequency (Nyquist)

– low-pass filtering of intensity-based data reduces aliasing but produces:
  • rounded corners and/or fuzzy surfaces
  • loss of detail

– when shading, gradient estimators exaggerate aliasing artifacts

Observations

– if the surface is known or can be estimated during pre-processing, we would like to encode it into the volume

– to avoid aliasing, we should represent surfaces with a function that varies slowly across the surface
**Observations**

- if the surface is known or can be estimated during pre-processing, we would like to encode it into the volume

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**Distance Maps**

- at each sample point, store the signed distance to the nearest surface point

- this sampled distance field is a *distance map*

- sign indicates inside vs. outside
**Distance Maps**

- distance maps:
  - vary smoothly across surfaces
  - produce a linear field for planar surfaces
  - can be reconstructed as the zero-value iso-surface of the distance map
  - can locally estimate the surface normal by the gradient of the distance map

**Intensity-Based Data**

- intensity-based data changes abruptly at edges
- the gradient of intensity-based data varies significantly in direction and magnitude
**DistanceMap Data**

- The distance map varies smoothly across the edge.
- The gradient of the distance map accurately estimates the edge normal.

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**Rendering from DistanceMaps**

For each sample point, we can store distance and intensity:

- Distance values are used to locate surfaces and estimate surface normals for shading.
- Intensity is used to assign object colors and opacities.
Distance-based Shading
Distance Maps calculated from implicit functions

Distance Maps calculated from polygonal models
Distance-based Shading

low resolution sampling for smooth models

R = 30 voxels volume $64^3$
R = 3 voxels volume $8^3$
R = 2 voxels volume $8^3$
R = 1.5 voxels volume $8^3$

Related Work

– gray-scale shading
– anti-aliased voxelization methods
– voxel-based carving
– distance maps for other applications
  • robotics
    (e.g. Koditschek 1989)
  • offset surfaces
  • object blending, surface texturing
    (Payne and Toga 1992)
**Generating Distance Maps**

- **analytic functions**
- **parametric (e.g., triangle) models**
- **binary data**

- store the signed distance to the nearest surface point at each volume sample point

- requires knowledge of the object surface

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\[ \text{dist}_{\text{sphere}} = R - (x^2 + y^2 + z^2)^{1/2} \]

\[ \text{dist}_{\text{torus}} = r - (x^2 + y^2 + z^2 + R^2 - 2R (x^2 + y^2)^{1/2})^{1/2} \]
Generating Distance Maps

- distance maps require a model of the underlying surface
- binary data has no explicit surface representation:
  → the surface must be estimated from the data

\[ \text{dist}_{\text{face}} = (p_v - v_0) \cdot n \]
\[ \text{dist}_{\text{vertex}} = \| p_v - v_0 \| \]
\[ \text{dist}_{\text{edge}} = \| u \times (p_e - v_0) \| \]
**Binary Models**

- Knee bones, volume rendered resolution: 0.27x0.27x1.4 mm
- Medial meniscus, polygon rendered resolution: 0.27x0.27x0.25 mm

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**Smoothing Binary Surfaces**

**Local smoothing**
- low pass filter the binary data
- use Marching Cubes to construct the surface model from the blurred data

**Global smoothing**
- warp a generalized parameterized surface to fit the surface of the binary data
Local Filtering

- analytic functions
- triangle models
- binary data

local filters:
- remove fine detail indiscriminately
- do not remove terracing artifacts

Global Filtering

- analytic functions
- triangle models
- binary data

global filters:
- choosing the parameterization is hard
- global tradeoffs between smoothness and surface detail
*SurfaceNets*

*Produces a globally smooth surface that preserves local detail*  

**Process:**

- initialize an elastic net on the object surface with nodes centered in each surface voxel
- relax the net to smooth the surface
- constrain nodes of the SurfaceNet so that they remain close to the binary object surface
SurfaceNets
**SurfaceNets Algorithm**

1) locate the surface cells

2) place a SurfaceNet node at the center of each surface cell

3) establish links between 6-connected surface nodes

4) adjust node positions to relax the SurfaceNet

5) constrain nodes to remain within their original surface cells

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**Generating Distance Maps**

- construct the triangle model from the surface net
- calculate the distance map from the triangle model
3D Examples
binary sphere, radius 30 voxels

original surface 1 relaxation 15 relaxations

3D Examples
binary sphere, radius 30 voxels

binary data
SurfaceNets
DistanceMap
volume rendering
image

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3D Examples

segmented femur

original surface 1 relaxation 12 relaxations

3D Examples

segmented femur

binary data

SurfaceNets

DistanceMap

volume rendering

image
Examples

segmented brain atlas

original SurfaceNet  relaxed SurfaceNet

segmented data courtesy of SPL
Brigham and Women's Hospital

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Examples

segmented brain atlas

binary data

SurfaceNets

DistanceMap

volume rendering

image

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Choosing Sampling Rates

Moller et al. (1997) observed that:

1) interpolation and gradient filters should depend on the sampled data

2) if the data is sampled from a linear field, we get exact reconstruction using:
   – tri-linear interpolation
   – central differences gradient estimator

– if the surface is planar, the distance map is a linear field

– hence, planar surfaces can be reconstructed exactly from a sampled distance map using linear filters

– using distance maps, sampling rates are based on surface curvature rather than the presence of surfaces
Choosing Sampling Rates

locally continuous and differentiable surfaces

\[ \text{dist}(p_1) = (p_1 - c_1) \cdot \hat{n}_1 \]
\[ \text{dist}(p_2) = (p_2 - c_2) \cdot \hat{n}_2 \]
\[ \quad = (p_2 - c_1) \cdot \hat{n}_1 + (p_2 - c_2) \cdot (\hat{n}_2 - \hat{n}_1) - (c_2 - c_1) \cdot \hat{n}_1 \]

The last two terms are small when the curvature is small and \(c_1\) and \(c_2\) are close together.

C1 Discontinuities

surface folds and corners

discontinuities:
- along A, B, and C

non-linear distance field:
- between D and E
Detecting Discontinuities

– for a linear distance field, the gradient magnitude is constant

– can detect singularities or discontinuities when the gradient magnitude is significantly different from this constant

→ in these cases higher order filters or other techniques can be used for reconstructing surfaces (e.g. Tiede et al)
Summary

Distance maps encode surfaces into volume data for high quality shading:

• they vary smoothly across surfaces
• the surface can be reconstructed as an iso-surface of the distance map
• the gradient of the distance map estimates local surface normal
• surface singularities and discontinuities can be dealt with