cs242

TYPES

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Reading: "Concepts in Programming Languages", Chapter 6

Thanks to John Mitchell for some of these slides.

Announcement

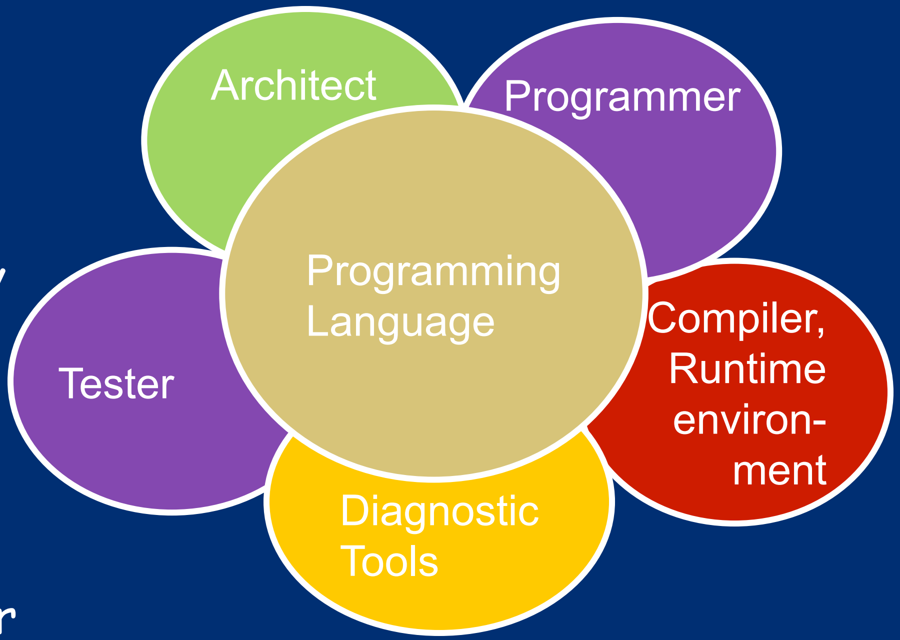
- We are looking for homework graders.
  - If you are interested, send mail to cs242@cs.stanford.edu
  - Need to be available approximately 5-9pm on Thursdays.
- You'll be paid Stanford's hourly rate.
- We'll provide food "of your choice."
- Previous graders have really enjoyed it.
- Great way to *really* learn the material.

Outline

- General discussion of types
  - What is a type?
  - Compile-time vs run-time checking
  - Conservative program analysis
- Type inference
  - Will study algorithm and examples
  - Good example of static analysis algorithm
- Polymorphism
  - Uniform vs non-uniform impl of polymorphism
  - Polymorphism vs overloading

Language Goals and Trade-offs

- Thoughts to keep in mind
  - What features are convenient for programmer?
  - What other features do they prevent?
  - What are design tradeoffs?
    - Easy to write but harder to read?
    - Easy to write but poorer error messages?
  - What are the implementation costs?



Type

A type is a collection of computable values that share some structural property.

- Examples

```
Integer
String
Int → Bool
(Int → Int) → Bool
```

- Non-examples

```
{3, True, \x->x}
Even integers
{f:Int → Int | if x>3
  then f(x) > x *(x+1)}
```

Distinction between sets that are types and sets that are not types is *language dependent*.

Uses for Types

- Program organization and documentation
  - Separate types for separate concepts
    - Represent concepts from problem domain
  - Indicate intended use of declared identifiers
    - Types can be checked, unlike program comments
- Identify and prevent errors
  - Compile-time or run-time checking can prevent meaningless computations such as 3 + true – “Bill”
- Support optimization
  - Example: short integers require fewer bits
  - Access record component by known offset

Compile-time vs Run-time Checking

- JavaScript and Lisp use run-time type checking

  f(x) *Make sure f is a function before calling f.*

```
js> var f= 3;
js> f(2);
typein:3: TypeError: f is not a function
js>
```

- ML and Haskell use compile-time type checking

  f(x) Must have $f : A \rightarrow B$ and $x : A$

- Basic tradeoff
  - Both kinds of checking prevent type errors.
  - Run-time checking slows down execution.
  - Compile-time checking restricts program flexibility.

    JavaScript array: elements can have different types

    Haskell list: all elements must have same type
  - Which gives better programmer diagnostics?

Expressiveness

- In JavaScript, we can write a function like

```
function f(x) { return x < 10 ? x : x(); }
```

  Some uses will produce type error, some will not.

- Static typing always conservative

```
if  (big-hairy-boolean-expression)
        then  f(5);
        else  f(15);
```

Cannot decide at compile time if run-time error will occur!

Relative Type-Safety of Languages

- Not safe: BCPL family, including C and C++
  - Casts, pointer arithmetic
- Almost safe: Algol family, Pascal, Ada.
  - Dangling pointers.
    - Allocate a pointer p to an integer, deallocate the memory referenced by p, then later use the value pointed to by p.
    - No language with explicit deallocation of memory is fully type-safe.
- Safe: Lisp, Smalltalk, ML, Haskell, Java, JavaScript
  - Dynamically typed: Lisp, Smalltalk, JavaScript
  - Statically typed: ML, Haskell, Java

Type Checking vs. Type Inference

- Standard type checking:

```
int f(int x) { return x+1; };
int g(int y) { return f(y+1)*2; };
```

  - Examine body of each function. Use declared types to *check* agreement.

- Type inference:

```
int f(int x) { return x+1; };
int g(int y) { return f(y+1)*2;};
```

  - Examine code *without* type information. *Infer* the most general types that could have been declared.

ML and Haskell are *designed* to make type inference feasible.

Why study type inference?

- Types and type checking
  - Improved steadily since Algol 60
    - Eliminated sources of unsoundness.
    - Become substantially more expressive.
  - Important for modularity, reliability and compilation
- Type inference
  - Reduces syntactic overhead of expressive types
  - Guaranteed to produce *most general type*.
  - Widely regarded as important language innovation
  - Illustrative example of a flow-insensitive static analysis algorithm

History

- Original type inference algorithm was invented by Haskell Curry and Robert Feys for the simply typed lambda calculus in 1958.
- In 1969, Hindley extended the algorithm to a richer language and proved it always produced the *most general type*.
- In 1978, Milner independently developed equivalent algorithm, called algorithm W, during his work designing ML.
- In 1982, Damas proved the algorithm was *complete*.
- Already used in many languages: ML, Ada, Haskell, C# 3.0, F#, Visual Basic .Net 9.0, and soon in: Fortress, Perl 6, C++0x
- We'll use ML to explain the algorithm because it is the original language to use the feature and is the simplest place to start.

ML Type Inference

- Example

  ```
  - fun f(x) = 2 + x;
  > val it = fn : int → int
  ```

- What is the type of f?
  - `+` has two types: `int → int → int`,
    `real → real → real`
  - `2` has only one type: `int`
  - This implies `+ : int → int → int`
  - From context, we need `x:int`
  - Therefore `f(x) = 2+x` has type `int → int`

Systematic Presentation

- Example

```
-fun f(x) = 2+x;
>val it = fn:int → int
```

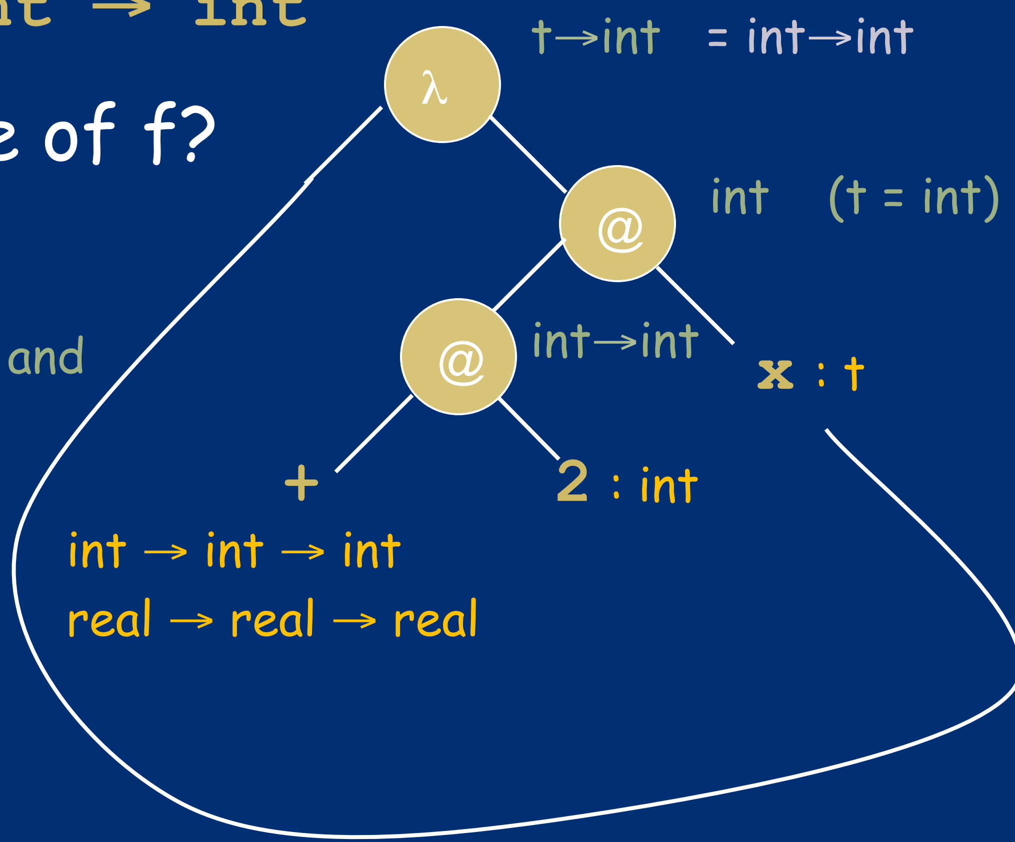
- What is the type of f?

Assign types to leaves

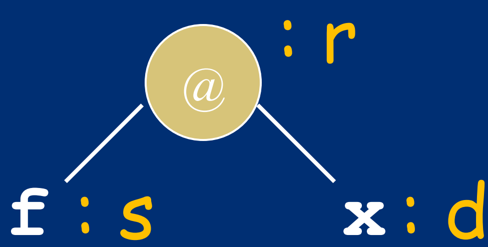
Propagate to internal nodes and generate constraints

Solve by substitution

Graph for `\x ->((plus 2) x)`



Application



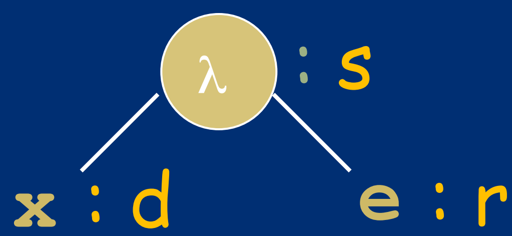
(s = domain → range)

(domain = d)

(range = r)

- Apply function f to argument x: `f(x)`
  - Because `f` is being applied, its type (s in figure) must be a function type: domain → range.
  - Domain of `f` must be type of argument x (d in figure).
  - Range of `f` must be result type of expression (r in figure).
  - Solving, we get: $s = d \rightarrow r$.

Abstraction



(s = domain → range)

(domain = d)

(range = r)

- Function expression: `\x -> e`
  - Type of lambda abstraction (s in figure) must be a function type: domain → range.
  - Domain is type of abstracted variable **x** (d in figure).
  - Range is type of function body **e** (r in figure).
  - Solving, we get : s = d → r.

Types with Type Variables

- Example

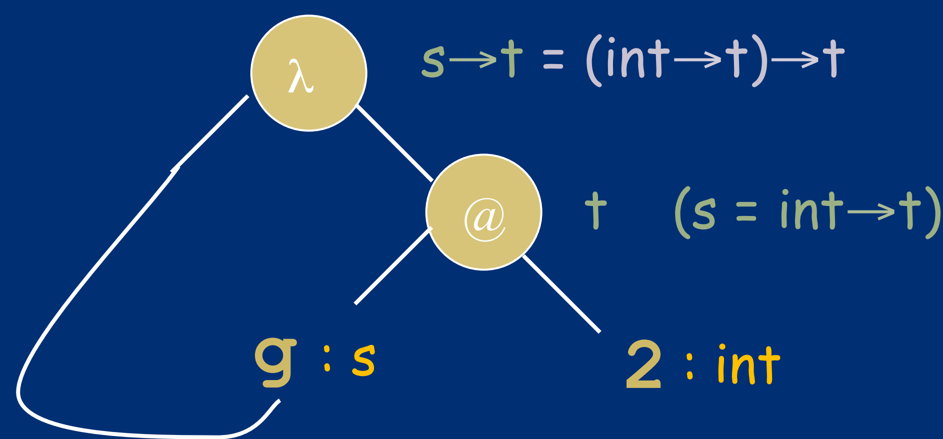
```
-fun f(g) = g(2);
>val it = fn : (int → t) → t
```

- What is the type of f?

Assign types to leaves

Graph for \g → (g 2)

Propagate to internal nodes and generate constraints



Solve by substitution

Use of Polymorphic Function

- Function

```
-fun f(g) = g(2);
>val it = fn:(int → t) → t
```

- Possible applications

```
-fun add(x) = 2+x;
>val it = fn:int → int
-f(add);
>val it = 4 : int
```

```
-fun isEven(x) = ...;
>val it = fn:int → bool
-f(isEven);
>val it = true : bool
```

Recognizing Type Errors

- Function

```
-fun f(g) = g(2);
>val it = fn:(int → t) → t
```

- Incorrect use

```
-fun not(x) = if x then false else true;
>val it = fn : bool → bool
-f(not);
Error: operator and operand don't agree
  operator domain: int -> 'Z
  operand:         bool -> bool
```

Type error: cannot make bool → bool = int → t

Another Type Inference Example

- Function Definition

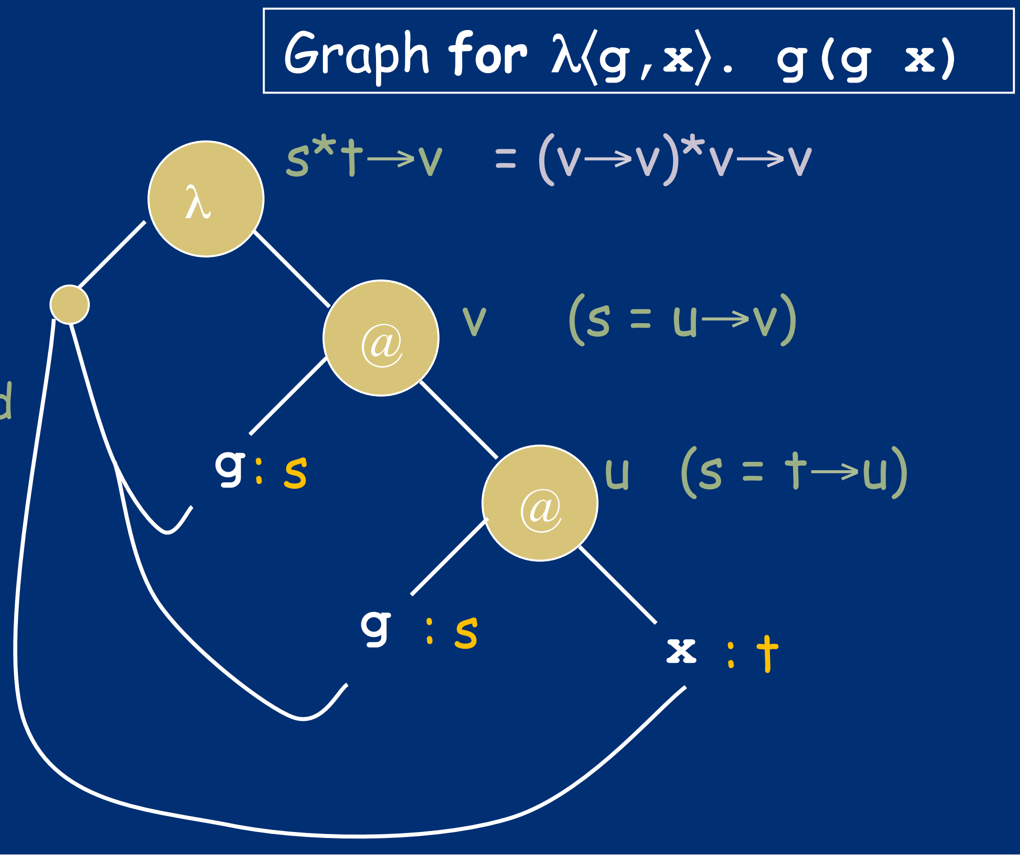
```
-fun f(g,x) = g(g(x));
>val it = fn:(t → t)*t → t
```

- Type Inference

Assign types to leaves

Propagate to internal nodes and generate constraints

Solve by substitution



Polymorphic Datatypes

- Datatype with type variable

```
- datatype 'a list = nil | cons of 'a *('a list)
> nil  : 'a list
> cons : 'a *('a list) → 'a list
```

'a is syntax for "type variable a"

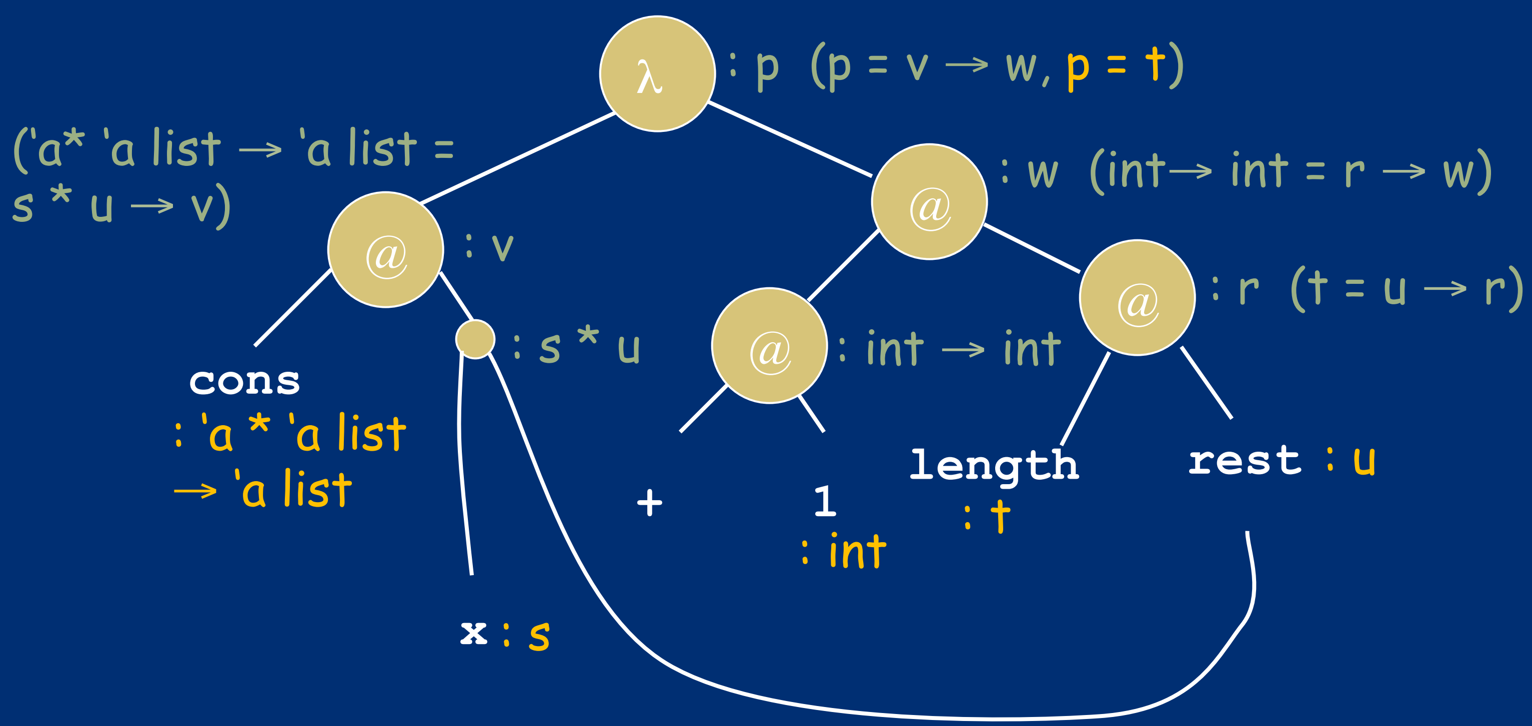
- Polymorphic function

```
- fun length nil = 0
    | length (cons(x,rest)) = 1 + length(rest)
> length : 'a list → int
```

- Type inference
  - Infer separate type for each clause
  - Combine by making two types equal (if necessary)

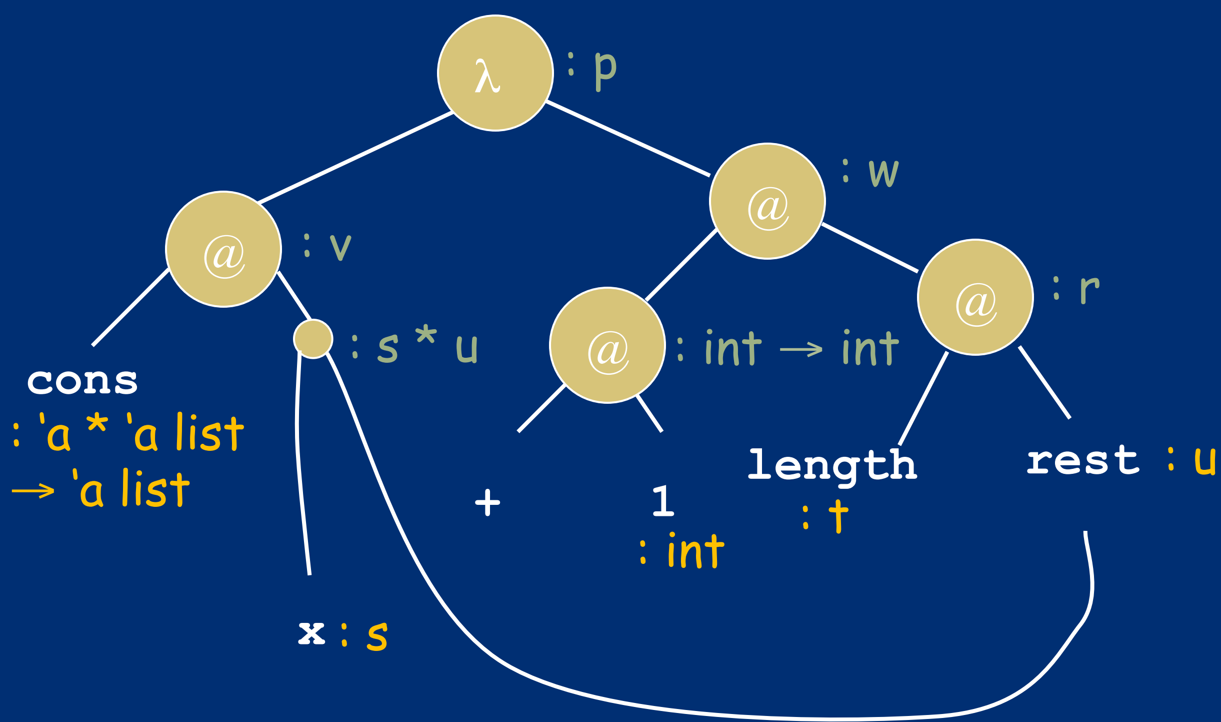
Type Inference with Recursion

- `length(cons(x,rest)) = 1 + length(rest)`



Type Inference with Recursion

- `length(cons(x,rest)) = 1 + length(rest)`



Collected Constraints:

p = t

p = v → w

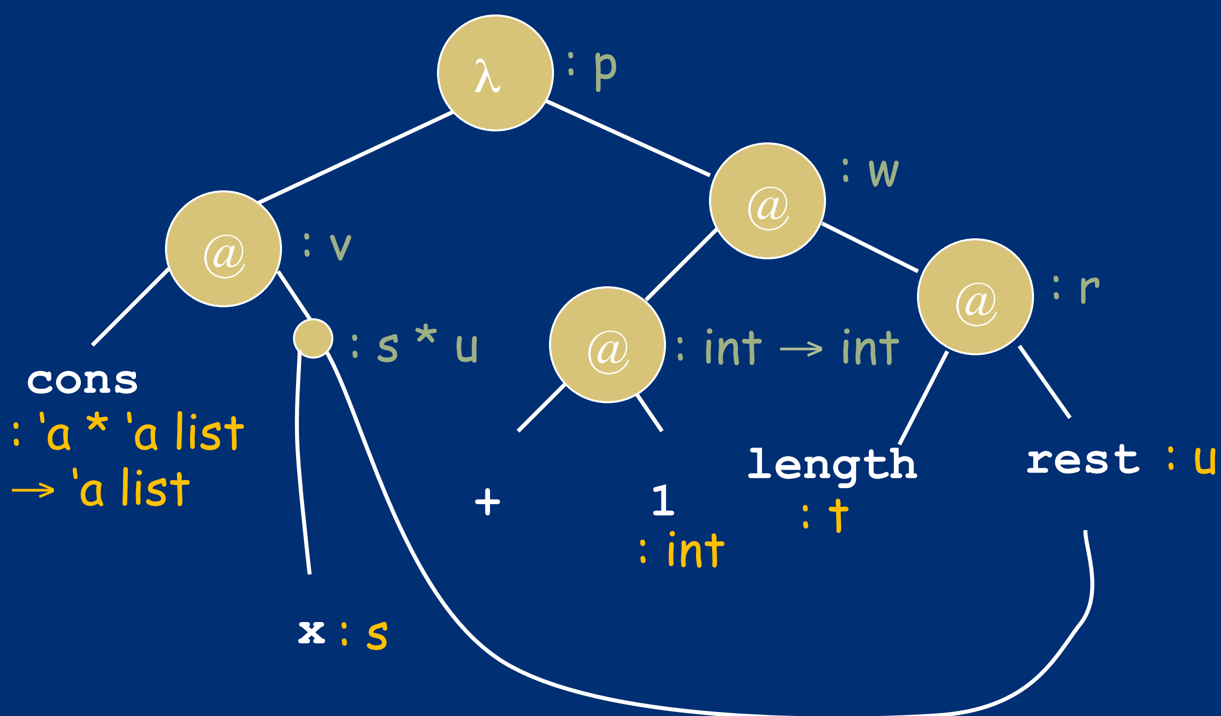
int→ int = r → w

t = u → r

$\boxed{\text{'a* 'a list → 'a list = s * u → v}}$

Type Inference with Recursion

- `length(cons(x,rest)) = 1 + length(rest)`



Collected Constraints:

p = t

p = v → w

int→ int = r → w

t = u → r

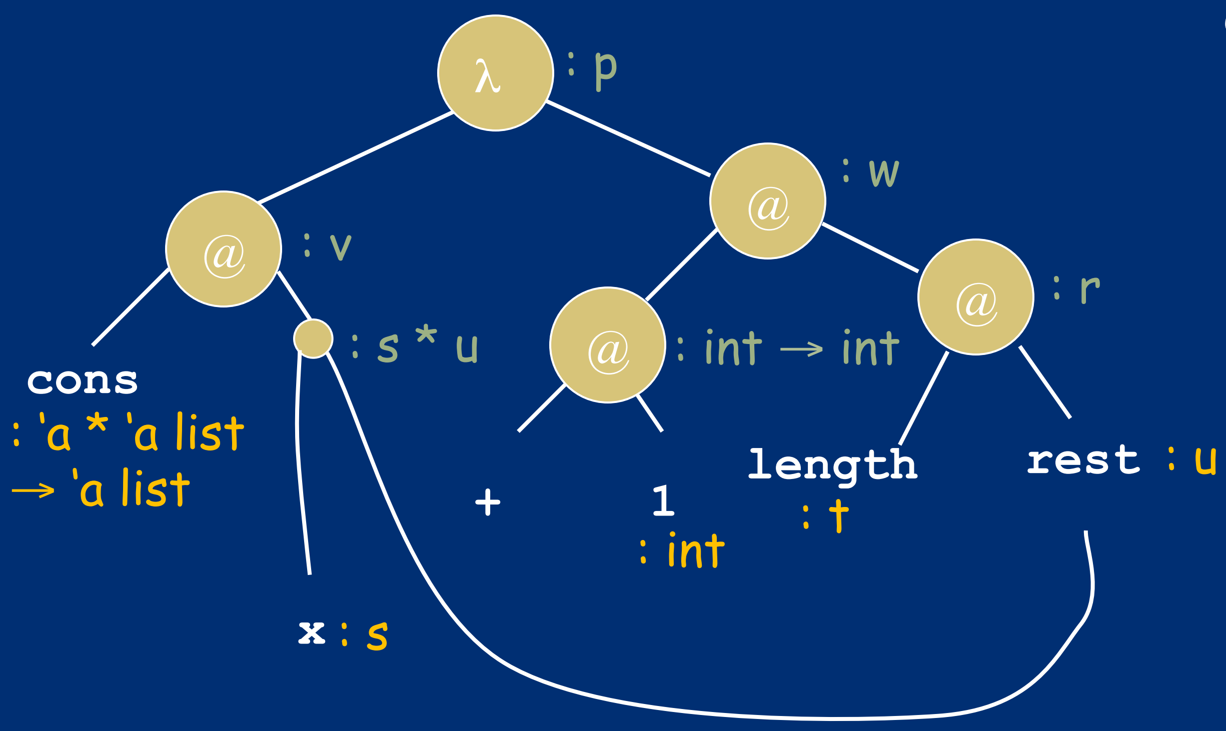
'a = s

'a list = u

'a list = v

Type Inference with Recursion

- `length(cons(x,rest)) = 1 + length(rest)`



Collected Constraints:

p = t

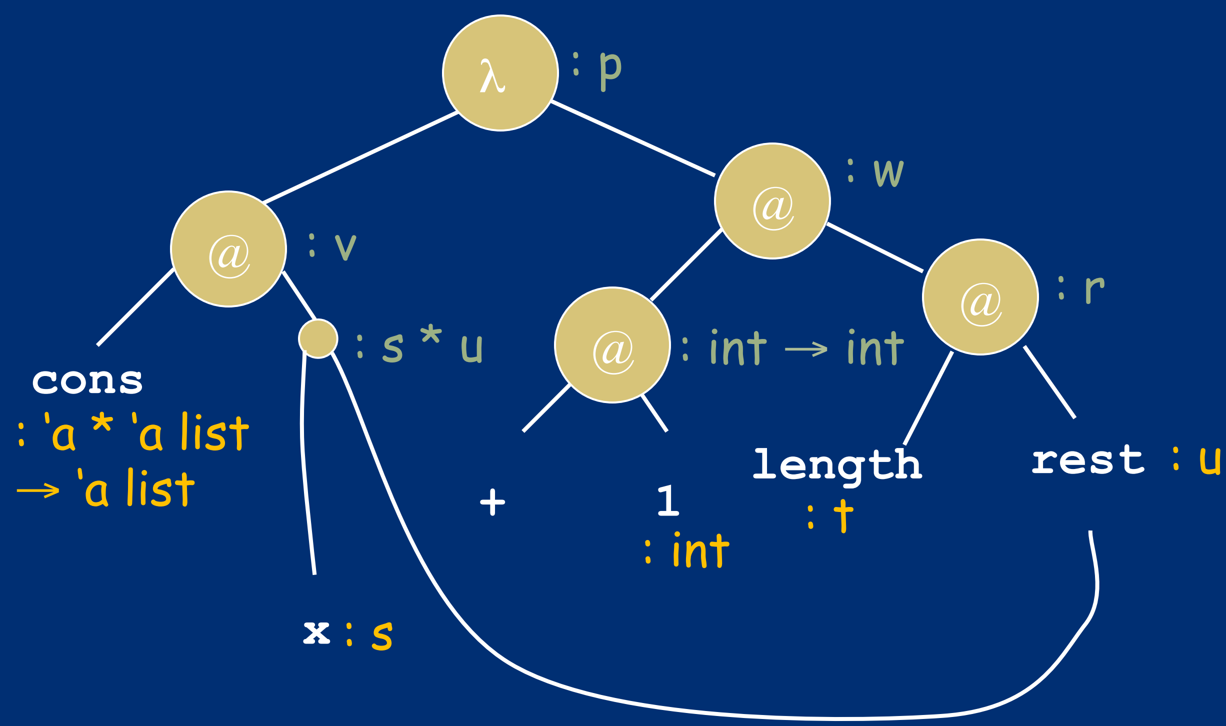
p = 'a list → w

$\boxed{\text{int} \to \text{int} = r \to w}$

t = 'a list → r

Type Inference with Recursion

- `length(cons(x,rest)) = 1 + length(rest)`



Collected Constraints:

p = t

p = 'a list → int

t = 'a list → int

Result:

p = 'a list → int

Multiple Clauses

- Function with multiple clauses

```
- fun append(nil,l) = l
    | append(x::xs,l) = x :: append(xs,l)
> append: 'a list * 'a list → int
```

- Infer type of each branch
  - First branch:

    `append :'a list *'b → 'b`

  - First branch:

    `append :'a list *'b → 'a list`

- Combine by equating types of two branches:

  `append :'a list *'a list → 'a list`

Most General Type

- Type inference is *guaranteed* to produce the *most general type*:

```
- fun map(f,nil) = nil
    | map(f, x::xs) = f(x) :: (map(f,xs))
> map:('a → 'b) * 'a list → 'b list
```

- Function has many other, less general types:
  - `map:('a → int) * 'a list → int list`
  - `map:(bool → 'b) * bool list → 'b list`
  - `map:(char → int) * char list → int list`
- Less general types are all *instances* of most general type, also called the *principal type*.

Type Inference Algorithm

- When the Hindley/Milner type inference algorithm was developed, its complexity was unknown.
- In 1989, Mairson proved that the problem was exponential-time complete.
- Tractable in practice though…

Information from Type Inference

- Consider this function…

```
fun reverse (nil) = nil
  | reverse (x::xs) = reverse(xs);
```

- … and its most general type:

```
reverse : 'a list → 'b list
```

- What does this type mean?

  Reversing a list does not change its type, so there must be an error in the definition of `reverse`!

See Koenig paper on "Reading" page of CS242 site

Type Inference: Key Points

- Type inference computes the types of expressions
  - Does not require type declarations for variables
  - Finds the *most general type* by solving constraints
  - Leads to polymorphism
- Sometimes better error detection than type checking
  - Type may indicate a programming error even if no type error.
- Some costs
  - More difficult to identify program line that causes error
  - ML requires different syntax for integer 3, real 3.0.
  - Natural implementation requires uniform representation sizes.
  - Complications regarding assignment took years to work out.
- Idea can be applied to other program properties
  - Discover properties of program using same kind of analysis

Haskell Type Inference

- Haskell also uses Hindley Milner type inference.
- Haskell uses type classes to support user-defined overloading, so the inference algorithm is more complicated.
- ML restricts the language to ensure that no annotations are required, ever.
- Haskell provides various features like *polymorphic recursion* for which types cannot be inferred and so the user must provide annotations.

Parametric Polymorphism: ML vs C++

- ML polymorphic function
  - Declarations require no type information.
  - Type inference uses type variables to type expressions.
  - Type inference substitutes for variables as needed to instantiate polymorphic code.
- C++ function template
  - Programmer must declare the argument and result types of functions.
  - Programmers must use explicit type parameters to express polymorphism.
  - Function application: type checker does instantiation.

ML also has module system with explicit type parameters

Example: Swap Two Values

- ML

```
- fun swap(x,y) =
  let val z = !x in x := !y; y := z end;
  val swap = fn : 'a ref * 'a ref -> unit
```

- C++

```
template <typename T>
void swap(T& x, T& y){
      T tmp = x;  x=y;  y=tmp;
}
```

Declarations look similar, but compiled very differently

Implementation

- ML
  - `Swap` is compiled into one function
  - Typechecker determines how function can be used
- C++
  - `Swap` is compiled into linkable format
  - Linker duplicates code for each type of use
- Why the difference?
  - ML ref cell is passed by pointer. The local x is a pointer to value on heap, so its size is constant.
  - C++ arguments passed by reference (pointer), but local x is on the stack, so its size depends on the type.

Another Example

- C++ polymorphic sort function

```
template <typename T>
void sort( int count, T * A[count ] ) {
  for (int i=0; i<count-1; i++)
      for (int j=i+1; j<count-1; j++)
        if (A[j] < A[i]) swap(A[i],A[j]);
}
```

- What parts of code depend on the type?
  - Indexing into array
  - Meaning and implementation of <

Polymorphism vs Overloading

- Parametric polymorphism
  - Single algorithm may be given many types
  - Type variable may be replaced by *any* type
  - if `f:t→t` then `f:int→int`, `f:bool→bool`, ...
- Overloading
  - A single symbol may refer to more than one algorithm
  - Each algorithm may have different type
  - Choice of algorithm determined by type context
  - Types of symbol may be arbitrarily different
  - \+ has types `int*int→int`, `real*real→real`, *no others*

ML Overloading

- Some predefined operators are overloaded
- User-defined functions must have unique type

```
- fun plus(x,y) = x+y;
```

This is compiled to `int` or `real` function, not both

- Why is a unique type needed?
  - Need to compile code, so need to know which +
  - Efficiency of type inference
  - Aside: General overloading is NP-complete

    Two types, *true* and *false*

    Overloaded functions

    and : {*true* * *true* → *true*, *false* * *true* → *false*, …}

Summary

- Types are important in modern languages
  - Program organization and documentation
  - Prevent program errors
  - Provide important information to compiler
- Type inference
  - Determine best type for an expression, based on known information about symbols in the expression
- Polymorphism
  - Single algorithm (function) can have many types
- Overloading
  - One symbol with multiple meanings, resolved at compile time