Example cases for problem 15

Here are some sample cases for the inductive proof required in Exercise 15 of Ramsey and Kamin.

Consider the rule for `if`:

$$\frac{\langle e_1, \xi, \phi, \rho\rangle \Downarrow \langle v_1, \xi', \phi, \rho'\rangle \qquad v_1 \neq 0 \qquad \langle e_2, \xi', \phi, \rho'\rangle \Downarrow \langle v_2, \xi'', \phi, \rho''\rangle}{\langle \textsc{if}(e_1, e_2, e_3), \xi, \phi, \rho\rangle \Downarrow \langle v_2, \xi'', \phi, \rho''\rangle.} \qquad (\textsc{IfTrue})$$

By the induction hypothesis, we can evaluate $\langle e_1, \xi, \phi, \rho\rangle \Downarrow \langle v_1, \xi', \phi, \rho'\rangle$ using a stack, and the evaluation will pop ρ and push ρ' without making a copy of ρ. Because ρ does not appear anywhere else in the rule, it is never used again, so it is safe to pop it and throw it away. We can use the induction hypothesis again to show that the evaluation of e_2 can pop ρ' and push ρ'', and ρ' is not copied. Moreover, ρ' is not used in the rule after the evaluation of e_2.

Finally, we see that ρ'' is used only as part of the result of the rule. We can conclude, then, that when e_1 evaluates to a nonzero value, we can safely evaluate $\textsc{if}(e_1, e_2, e_3)$ on a stack, and the evaluation effectively pops ρ, which is never used again, then pushes ρ''.

The FormalVar rule is one of the base cases; it doesn't require the induction hypothesis.

$$\frac{x \in \operatorname{dom} \rho}{\langle \textsc{var}(x), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle} \qquad (\textsc{FormalVar})$$

By examining the rule, we see that it is possible to implement it as follows: pop ρ, test $x \in \operatorname{dom} \rho$, and compute $\rho(x)$. Then push ρ back on the environment stack, after which the only copy is once again on top of the stack.