Assignment 4

This assignment is due in class on Thursday 12/3.

Problem 1  Solve problem 13.1 page 147 in [SB]

Problem 2  Solve problem 13.3 page 148 in [SB]

Problem 3

We have an algorithm which has access to \( n \) “advisors” (think of an advisor as a variable or feature). On each example, some advisors make a prediction and some abstain. Based on these responses, the learning algorithm must make a prediction, after which it is told the correct answer, charged a penalty of 1 if it made a mistake, and then the process repeats. Let’s suppose that out of the \( n \) advisors, there is a subset of \( r \) “good” ones such that (i) none of the good ones ever produces an incorrect prediction, and (ii) on every example at least one good advisor predicts something.

We will use the following Winnow variant. Initialize all advisors to start with weight 1. Given an example, poll the advisors and produce the prediction that has the highest weighted vote. If a mistake is made, multiply the weights of advisors that predicted incorrectly by \( \frac{1}{2} \), multiply the weights of advisors that predicted correctly by \( \frac{3}{2} \), and leave those that abstained alone.

Prove that the maximum number of mistakes made by this algorithm is \( r \log_{3/2} \left( \frac{n}{r} \right) \).

*Hint:* Use the fact that the total weight of the good advisors cannot be larger than the total weight of all the advisors. How do these values change when weights are updated?

Problem 4

Read sections 7.1-2 and problem 7.2 in [KV]

Show that if halfspaces are learnable then the class of exclusive-or (XOR) of two halfspaces is also learnable.

*Notes:* (1) In your solution you may assume that, under the distribution \( D \), points on the boundary have zero probability. That is, any point \( x \) satisfying \( \sum u_i x_i = 0 \) or \( \sum v_i x_i = 0 \) has \( Pr(x) = 0 \). (2) Given a sample \( S \) that is correctly labeled by an XOR of halfspaces, find an appropriate instance transformation translating it into a sample \( S' \) (over some other feature space) which is correctly labeled by some halfspace.