150SRL, Computer Science, Tufts University, Spring 2009, Roni Khardon
Assignment 2

The assignment is due Tuesday March 10 in class.

1 Herbrand Interpretations
In this part we consider a language with signature including predicate plus() taking 3 arguments, function zero() (using shorthand 0 below) with no arguments, and function successor() (using shorthand s(·) below) with one argument.

1. What is the Herbrand Universe of the expression \( \text{plus}(s(0), s(s(0)), s(s(s(0)))) \)?

2. What is the Herbrand Universe of the expression \( \forall x, \forall y, \forall z, \text{plus}(x, y, z) \rightarrow \text{plus}(s(x), y, s(z)) \)?

3. Give one Herbrand Interpretation which is a model of the previous rule.

4. Give one Herbrand Interpretation which is not a model of the rule.

2 Logic and Inference with the Resolution Operator

1. Formalize the following English statements in logic:
   - If someone is a child of at least one dragon then it is a dragon.
   - A dragon is green if it is a child of at least one green dragon.
   - Green dragons can fly.
   - A dragon is happy if all its children can fly.
   - All green dragons are happy.
   
   In your formalization use predicates \( \text{dragon}(X) \), \( \text{green}(X) \), \( \text{happy}(X) \), \( \text{fly}(X) \), \( \text{child}(X,Y) \).

2. Convert the expressions into Skolemized normal form with CNF as formula, to get a set of universally quantified disjunctions.

3. Use resolution to give a refutation proof showing that the last expression (All green dragons are happy) follows from the previous ones. To do this negate and skolemize the last expression and use it with the previous ones to derive a contradiction.

3 What to submit

Please submit hardcopies in class.