Convolution Surfaces
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Skeletal Methods of Shape Manipulation
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Convolution Surfaces

- Need for smooth, flexible solids and surfaces
- Splines, Bezier curves
- Implicit surfaces
- Convolution surfaces: Simple skeleton controls the shape of a more complicated object
Potential Surfaces

\[ f(S, p) = \sum_{s \in S} \exp\left(\frac{-\|s-p\|^2}{2}\right). \]

- Implicit function from set \( S \) of center points
- \( p \) is any point such that \( f(S, p) - c = 0 \) where \( c \) is the iso-potential value (inside/outside threshold)
- Spherical function of 3D distance from point
- Non negative \( f() \) is interior of volume (set operations possible)
- Smooth merging of multiple points
Distance Surfaces

\[ f(S, p) = \max_{s \in S} \exp\left(-\frac{\| s-p \|^2}{2}\right) \]

- Replace sum from potential surfaces with max
Convolution Surfaces

\[ f(S, p) = \int_S \exp \left( \frac{-\| s - p \|^2}{2} \right) \, ds \]

\[ h(p) = \exp \left( \frac{-\| p \|^2}{2} \right) \]

\[ f(p) = (h \ast S)(p) = \int_S \exp \left( \frac{-\| s - p \|^2}{2} \right) \, ds \]

- Integral is sum of infinitesimal potential surface points
- Star is convolution
- Like a potential surface integrated along a skeletal line
Visualization

Skeleton  Convolution  Distance
Superposition

\[ h \star (S_1 + S_2) = (h \star S_1) + (h \star S_2). \]
Implementation (Rendering)

$$f(S, p) = \exp\left(\frac{-||z_s - z_p||^2}{2}\right) \int_{S_y} \exp\left(\frac{-||y - y_p||^2}{2}\right) \int_{S_x} \exp\left(\frac{-||x - x_p||^2}{2}\right) dx \, dy$$
Variations: 5 Ways to Change Shape

• Change iso-potential value
• Change skeleton shape (obviously)
• Skeleton weight (surface height, negative = pit)
• Change Gaussian (field) width
• Deform skeleton (non-planar)
Deformation Example

- Twisted rectangle skeleton
Convolution Surface Summary

- Skeleton shape determines surface shape
- Smooth surface regardless of skeleton
- Surface varies smoothly with skeleton
- Field width allows local control
- Multiple surfaces blend well
- Fast implementation
- No geometric surface is generated
Skeletal Methods of Shape Manipulation

- Go from object to inverse kinematic (IK) skeleton
- Object may be scanned, procedurally created, or hand sculpted
- Change the IK skeleton
- Go back to object
Overview

IK skeleton  Geometric skeleton  Surface
Skeletonization: Old Ways

• Delaunay Triangulation
• Angle Bisection
• Volumetric Thinning
• All have limitations, so authors invented Direction Testing
Skeletonization: New Way

- Direction Testing: skeleton is set of points at which direction to the nearest point on the object undergoes a sudden transition
Non-Manifold Polygonization
Go Fast

- n nearest triangles assigned to octree terminal node centered at \( c \) such that \( d_n - d_1 > \) length of node’s major diagonal
- Claim linear time
Reconstruction

• Every point on geometric skeleton has a distance and angle to a point on IK skeleton
• When IK skeleton is modified, geometric skeleton is modified “accordingly”
• Create distance surface from geometric skeleton
• Polygonize (make a mesh)
• Optionally use convolution surfaces to reconstruction concavities
Conclusions

• Skeletons are ideal representation of animated objects
• Can be created automatically from procedurally modeled or scanned objects
• Compact storage
• Easy for designers and artists to make smooth rounded objects
Future Work

• What is the “language” of skeletal modeling?
• What is the optimal form of user interaction?