Example class `Fraction`: initialization

```
(class Fraction Number
    [num den] ;; representation (concrete!)
              ;; invariants by signReduce, divReduce
    (class-method num:den: (a b)
        (initNum:den: (new self) a b))
    (method initNum:den: (a b) ; private
        (setNum:den: self a b)
        (signReduce self)
        (divReduce self))
    (method setNum:den: (a b)
        (set num a) (set den b) self) ; private
    .. other methods of class Fraction ...
)
```

Information revealed to self

"Instance variables" `num` and `den`

- Directly available
- Always and only go with `self`

Object knows its own representation, invariants, private methods:

```
(method asFraction ()
   self)
(method print ()
   (print num) (print '/) (print den))
(method reciprocal ()
   (signReduce (setNum:den: (new Fraction) den num)))
```

Information revealed to self: your turn

How would you implement `coerce:`?
(Value of argument, representation of receiver)

```
(method asFraction ()
   self)
(method print ()
   (print num) (print '/) (print den))
(method reciprocal ()
   (signReduce (setNum:den: (new Fraction) den num)))
(method coerce: (aNumber)
   ...)
```

Information revealed to self: your turn

How would you implement `coerce:`?
(Value of argument, representation of receiver)

```
(method asFraction ()
   self)
(method print ()
   (print num) (print '/) (print den))
(method reciprocal ()
   (signReduce (setNum:den: (new Fraction) den num)))
(method coerce: (aNumber)
   (asFraction aNumber))
```

Exposing information, part II

Alas! **Cannot see representation of argument**

How will you know “equal, less or greater”?

Exposing information, part II

Alas! Cannot see representation of argument

Protocol says "like with like"? Use private methods

```
(method num () num) ; private
(method den () den) ; private

(method = (f)  ;; relies on invariant!
    (and: (= num (num f)) {(= den (den f))}))
(method < (f)
    (< (* num (den f)) (* (num f) den)))
```

Remember behavioral subtyping

Private methods: Your turn

How will you multiply two fractions?

Private methods: Your turn

How will you multiply two fractions?

```
(method * (f)
    (divReduce
        (setNum:den: (new Fraction)
                        (* num (num f))
                        (* den (den f)))))
```

An open system

`Number` protocol: like multiplies with like

What about large and small integers?

- How to multiply two small integers?
- How to multiply two large integers?

How is algorithm known?

Each object knows its own algorithm:

- Small: Use machine-primitive multiplication
- Large: Multiply magnitudes; choose sign

Review: Two kinds of knowledge

I can send message to you:

- I know your protocol

I can inherit from you:

- I know my subclass responsibilities

Knowledge of protocol

Three levels of knowledge:

1. I know only your public methods
   Example: send `select:` to any collection
2. You are like me: share private methods
   Example: send `*` or `+` to `Fraction`
3. I must get to know you: double dispatch
   Example: send `*` to `+` to any integer

Double dispatch: extending open systems

I claim:

- **Large integers and small integers both `Integer`**
- **Messages `=`, `<`, `+`, `*` ought to mix freely**
- **Large and small integers have different private protocol**

Private for large integers: `magnitude`

Private for small integers: `mul:withOverflow`

Double dispatch: forms of argument

Many kinds of multiplication:

```
(:+ n) * (:- m) ==  :- (n * m)
(:+ n) * (:+ m) ==  :+ (n * m)
(:+ n) * small  ==  (:+ n) * (asLargeInteger small)
```

But! Can't distinguish forms of argument

Solution: "dispatch laws"

```
(:+ n) * (:- m) ==  (timesLP: (:- m) self)
(:+ n) * (:+ m) ==  (timesLP: (:+ m) self)
(:+ n) * small  ==  (timesLP: small  self)
```

Argument to `timesLP:`

- **Understands "large positive integer" protocol**

Double dispatch codes operation & protocol

Example messages:

- I answer the large-positive integer protocol, multiply me by yourself
- I answer the small-integer protocol, add me to yourself

Message encodes

- Operation to be performed
- Protocol accepted by argument

Your turn: responding to double dispatch

How do you act?

1. **As small integer, you receive "multiply large positive integer N by `self`"**
2. **As small integer, you receive "add small integer n to `self`"**
3. **As large positive integer, you receive "multiply large positive integer N by `self`"**
4. **As large positive integer, you receive "add small integer n to `self`"**

Your turn: using double dispatch

On what class does each method go?

A.
```
(method + (aNumber)
   (addSmallIntegerTo: aNumber self))
```
B.
```
(method * (anInteger)
   (multiplyByLargePositiveInteger: anInteger self))
```

(See the “double dispatch”: `+` then `addSmallIntegerTo:`)

Information-hiding summary

Three levels

1. I use your public protocol
2. We are alike; I add our private protocol
3. Your protocol is revealed by double dispatch

Extra: Dealing with overflow

New law for multiplication:

```
(* small-1 small-2) =
  (mulSmall:withOverflow:
     small-1
     small-2
     {(* (asLargeInteger small-1) small-2)})
```

Block is exception block run on overflow

Method is primitive, defined with

```
  (method mulSmall:withOverflow:
          primitive mul:withOverflow:)
```

Subtyping mathematically

Always transitive

$$\frac{\tau_1 <: \tau_2 \qquad \tau_2 <: \tau_3}{\tau_1 <: \tau_3}$$

Key rule is subsumption:

$$\frac{e : \tau \qquad \tau <: \tau'}{e : \tau'}$$

(*implicit* subsumption: no cast)

Subtyping is not inheritance

Subtype understands *more* messages:

$$\frac{}{\{m_1 : \tau_1, \ldots, m_n : \tau_n, \ldots, m_{n+k} : \tau_{n+k}\} <: \{m_1 : \tau_1, \ldots, m_n : \tau_n\}}$$

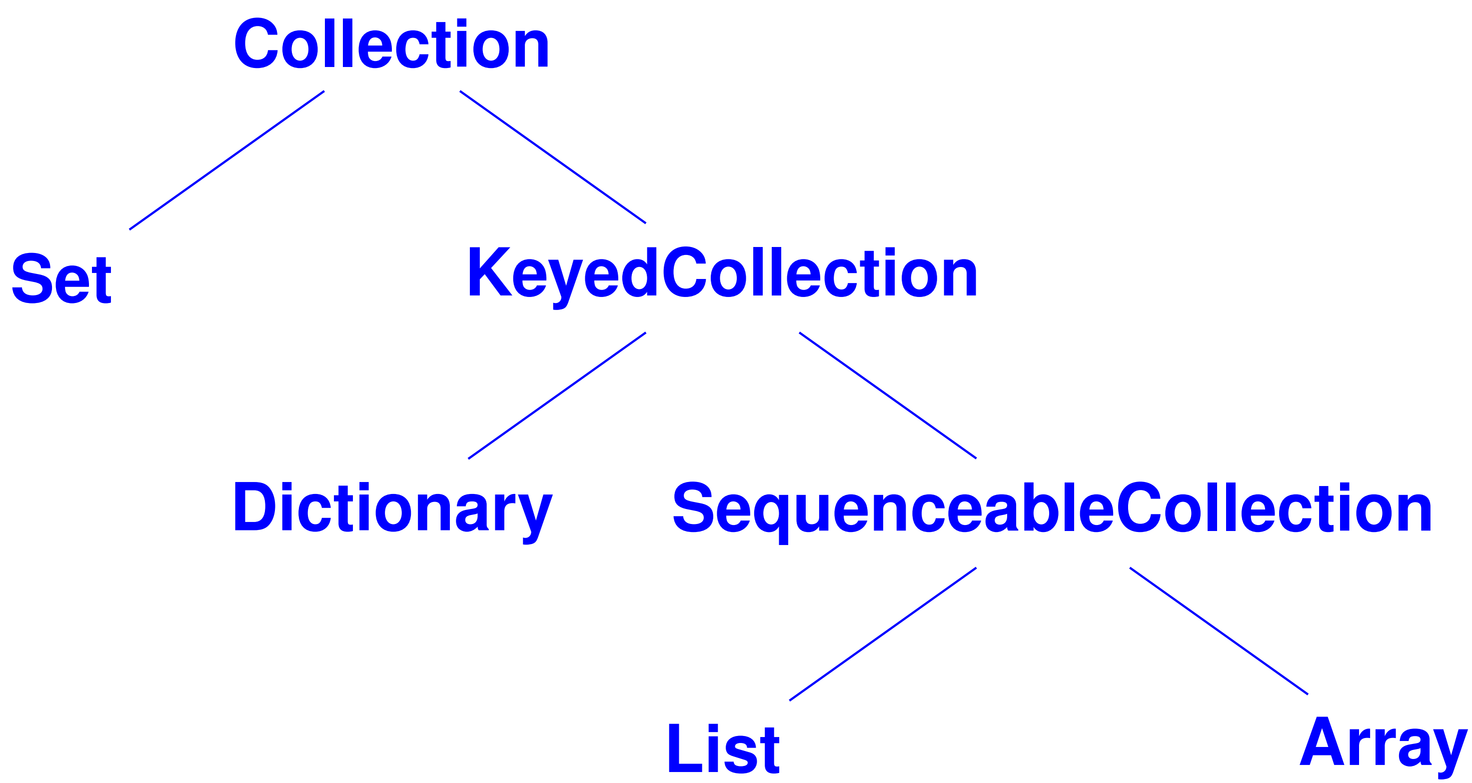
If an object understands messages $m_1, \ldots, m_n$, and possibly more besides, you can use it where $m_1, \ldots, m_n$ are expected

- **Methods must behave as expected**

Behavioral subtyping (in Ruby, "duck typing")

```
(class Set Collection
  [members] ; list of elements
  (class-method new () (initSet (new super)))
  (method initSet () ; private method
     (set members (new List))
     self)
  (method do: (aBlock) (do: members aBlock))
  (method remove:ifAbsent: (item exnBlock)
    (remove:ifAbsent: members item exnBlock))
  (method add: (item)
    (ifFalse: (includes: members item)
      {(add: members item)})
    item)
  (method species () Set)
  (method asSet   () self) ; extra efficient
)
```

“Collection hierarchy”



Collection mutators

`add: newObject` Add argument

`addAll: aCollection` Add every element of arg

`remove: oldObject` Remove arg, error if absent

`remove:ifAbsent: oldObject exnBlock`
Remove the argument, evaluate `exnBlock` if absent

`removeAll: aCollection` Remove every element of arg

Collection observers

`isEmpty` **Is it empty?**

`size` **How many elements?**

`includes: anObject` **Does receiver contain arg?**

`occurrencesOf: anObject` **How many times?**

`detect: aBlock` **Find and answer element satisfying** `aBlock` **(cf μScheme** `exists?`**)**

`detect:ifNone: aBlock exnBlock` **Detect, recover if none**

`asSet` **Set of receiver's elements**

Collection iterators

`do: aBlock` For each element `x`, evaluate `(value aBlock x)`.

`inject:into: thisValue binaryBlock`
Essentially μScheme `foldl`

`select: aBlock` Essentially μScheme `filter`

`reject: aBlock` Filter for *not* satisfying `aBlock`

`collect: aBlock` Essentially μScheme `map`

Implementing collections

```
(class Collection Object
  [] ; abstract
  (method do: (aBlock)
     (subclassResponsibility self))
  (method add: (newObject)
     (subclassResponsibility self))
  (method remove:ifAbsent (oldObj exnBlock)
     (subclassResponsibility self))
  (method species ()
     (subclassResponsibility self))
  ⟨other methods of class Collection⟩
)
```

Reusable methods

⟨*other methods of class* Collection⟩=

```
(method addAll: (aCollection)
   (do: aCollection [block(x) (add: self x)])
  aCollection)
(method size () [locals temp]
 (set temp 0)
 (do: self [block(_) (set temp (+ temp 1))])
 temp)
```

These methods always work

Subclasses can override (redefine) with more efficient versions

`species` method

Create "collection like the reciever"

Example: filtering

⟨*other methods of class* `Collection`⟩=

```
(method select: (aBlock) [locals temp]
  (set temp (new (species self)))
  (do: self [block (x)
     (ifTrue: (value aBlock x)
        {(add: temp x)})])
  temp)
```

```
(class Set Collection
  [members] ; list of elements
  (class-method new () (initSet (new super)))
  (method initSet () ; private method
     (set members (new List))
     self)
  (method do: (aBlock) (do: members aBlock))
  (method remove:ifAbsent: (item exnBlock)
    (remove:ifAbsent: members item exnBlock))
  (method add: (item)
    (ifFalse: (includes: members item)
      {(add: members item)})
    item)
  (method species () Set)
  (method asSet   () self) ; extra efficient
)
```