

## Lists defined inductively

$\text{LIST}(A)$  is the smallest set satisfying this equation:

$$\text{LIST}(A) = \{ ' () \} \cup \{ (\text{cons } a \text{ as}) \mid a \in A, \text{as} \in \text{LIST}(A) \}$$

Equivalently,  $\text{LIST}(A)$  is defined by these rules:

$$\frac{}{' () \in \text{List}(A)} \text{(EMPTY)}$$

$$\frac{a \in A \quad \text{as} \in \text{List}(A)}{(\text{cons } a \text{ as}) \in \text{List}(A)} \text{(CONS)}$$

# One more inductive definition

A list of  $A$  is one of:

- The empty list ' ()
- (cons a as), where a is an  $A$  and as is a list of  $A$

# **Lists generalized: S-expressions**

**An ordinary S-expression is one of:**

- An atom (symbol, number, Boolean)
- A list of ordinary S-expressions

**Can write literally in source, with quote**

# $\mu$ Scheme vs Impcore

New abstract syntax:

**LET (keyword, names, expressions, body)**

**LAMBDA(X (formals, body)**

**APPLY (exp, actuals)**

New concrete syntax for LITERAL:

(**quote** *S-expression*)

' *S-expression*

## Equations and function for append

(append ' () ys) == ys

(append (cons z zs) ys) == (cons z (append zs ys))

(define append (xs ys)

(if (null? xs)

ys

(cons (car xs) (append (cdr xs) ys))))

## Naive list reversal

```
(define reverse (xs)
  (if (null? xs)
      ' ()
      (append (reverse (cdr xs))
              (list1 (car xs))))))
```

## Reversal by accumulating parameters

```
(define revapp (xs ys)
  ; return (append (reverse xs) ys)
  (if (null? xs)
      ys
      (revapp (cdr xs)
              (cons (car xs) ys)))))

(define reverse (xs) (revapp xs '()))
```

## A-list example

```
-> (find 'Building
      ' ((Course 105) (Building Barnum)
          (Instructor Ramsey)))
Barnum
-> (val nr (bind 'Office 'Halligan-222
                  (bind 'Courses '(105 150TW)
                      (bind 'Email 'comp105-grades '()))))
  ((Email comp105-grades)
   (Courses (105 150TW))
   (Office Halligan-222))
-> (find 'Office nr)
Halligan-222
-> (find 'Favorite-food nr)
()
```

## Laws of association lists

```
(find k (bind k v a-l)) = v  
(find k (bind k' v a-l)) = (find k a-l), provided k != k'  
(find k '()) = '() ---- bogus!
```