Induction and Recursion Using an Imperative Core

COMP 105 Assignment

Due Tuesday, January 30, 2018 at 11:59PM

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NOTE: This assignment is due one minute before midnight. You may turn it in up to 24 hours after the due date, which will cost you one extension token. If you wish not to spend an extension token, then when midnight arrives submit whatever you have. We are very willing to give partial credit.

ALERT: This assignment is three or four times easier than a typical COMP 105 assignment. Its role is to get you acclimated and to help you start thinking systematically about how recursion works. Later

1../syllabus.html#what-if-i-cant-get-my-homework-in-on-time
assignments get *much* harder and more time-consuming, so don’t use this one to gauge the difficulty of the course.

**Reading-Comprehension Questions (10%)**

Please read pages 6–14, in *Programming Languages: Build, Prove, and Compare*. Then place the answers to the following questions in a text file called `cqs.impcore.txt`. (You can download the questions.) Answer these questions before starting the other problems below.

1. What is the value of the following Impcore expression?
   \[(\text{if} \ (> \ 3 \ 5) \ 17 \ 99)\]

2. Which of the following best describes the syntactic structure of Impcore?
   (a) An expression can contain a definition
   (b) A definition can contain an expression
   (c) Both of the above
   (d) None of the above

3. Does the following Impcore test pass? Please answer “yes” or “no.”
   \[(\text{check-expect} \ (+ \ 1 \ 2 \ 3) \ 6)\]
   Assumining \(x\) is bound to a global variable, does the following Impcore test pass? Again, please answer “yes” or “no.”
   \[(\text{check-expect} \ (\text{set} \ x \ 1) \ 1)\]

Next read section 1.2, which starts on page 15, about abstract syntax.

4. After reading about abstract syntax, look at this picture of an abstract-syntax tree for a “calculator expression”:

   ![Abstract Syntax Tree](image)

   Answer these questions:
   (a) What concrete syntax could you write in C for this expression?

---

2`/cqs.impcore.txt`
(b) What concrete syntax could you write in Impcore for this expression?

Finally, read the handout on five proof systems for natural numbers.

5. Professor Ramsey gives you a recursive function \( f \) that takes one argument, a natural number \( n \). The recursion pattern of the function is based on the Peano proof system from the handout.

(a) For what values of \( n \) do you expect \( f \) to make a recursive call?
(b) When a recursive call is made, what value is passed as argument?

**Programming in Impcore (80%)**

The problems below are simple programming exercises that serve multiple purposes: to get you thinking explicitly about induction and recursion, to get you acclimated to the LISP-style concrete syntax used throughout the course, to get you started with the course software, and to help you practice the forms of testing and documentation that are expected in the course. You can start these exercises immediately after the first lecture. If you find it entertaining, you may write very efficient solutions—but do not feel compelled to do so. Just make sure that your recursive functions terminate!

**Do not share your solutions with anyone.** We encourage you to discuss ideas, but no other student may see your code.

**Getting set up with the software**

The interpreter you need to run Impcore code is provided as part of the course. To add course software to your execution path, run

```
use -q comp105
```

You may want to put this command in your .cshrc or your .profile. The -q option is needed to prevent use from spraying text onto standard output, which may interfere with with scp, ssh, git, and rsync.

The `impcore` interpreter is in /comp/105/bin; if you have run `use` as suggested above you should be able to run it just by typing

```
ledit impcore
```

The `ledit` command gives you the ability to retrieve and edit previous lines of input.

If your code and unit tests are in file solution.imp, you can load and run them by typing

```
impcore -q < solution.imp
```

**Unit testing**

The special “extended-definition forms” check-expect, check-assert, and check-error are part of every language in the book. For example, as described in section 1.1.1 of the book (page 6), they are part of the Impcore language. These forms serve both as unit tests and as documentation. **Every function you write must be tested and documented using check-expect or check-assert**, and possibly also check-error. The number of unit tests must be appropriate to the function’s contract and to the structure
of its input. In this first assignment, briefly explain the purpose of each check-expect or other unit tests, and explain why the tests are sufficient to exercise every subexpression in the function’s body.

How do you know when to use check-expect and when to use check-assert? Like this: check-expect tells if two expressions evaluate to the same value, and check-assert checks to see if a single expression evaluates to truth or falsehood. To be confident you are getting things right,

- If you are writing a check-expect that something is 0 or 1, meant as a Boolean, then probably you should be writing check-assert. (If checking for 0, remember to use not, like the examples in the lecture notes.)
- If you are writing a check-assert of an expression written with =, you should definitely write check-expect instead. It’s more readable, and if it fails, you get a more useful error message.

**Documentation**

In addition to its unit tests, each function should be documented by a contract which explains what the function does. Here’s an example:

```plaintext
;; (occurs-in? d n) returns 1 if and only if decimal digit ‘d’
;; occurs in the decimal representation of the positive integer
;; ‘n’; it returns 0 otherwise.
```

Expectations about contracts are explained in the course coding guidelines. Please read them. The contract is typically supplemented by unit tests, which can help clarify the contract:

```plaintext
(check-expect (occurs-in? 7 123) 0)
(check-expect (occurs-in? 2 123) 1)
```

**Connecting induction and recursion**

Study of programming languages exposes deep connections between induction and recursion: a recursive computation often has the same structure as an inductive proof. Our proof-systems handout presents five simple inductive proof systems, and it illustrates one connection between proof and recursion. In this assignment you will document such connections.

To document connections, please write, for each recursive function, which proof system from the handout, if any, corresponds to its pattern of recursion.

- If the recursion pattern corresponds to one of the proof systems in the handout, your documentation should say that it follows the “Peano”, “binary”, “decimal”, “decimal2”, or “parity” system.
- If the recursion pattern does not correspond to any of the proof systems in the handout, your documentation should say that “the recursion pattern is based on this idea about natural numbers,” then give your idea. An idea is a list of cases that constitute natural numbers, each one of which corresponds to an inference rule. For example, here is an idea you might find useful:

```
Zero and one are natural numbers. And if m and m + 1 are both natural numbers, so is m + 2.
```

---

3. ../coding-style.html#contracts
4. ../handouts/natproofs.pdf
Please document your recursion pattern below the function, not as part of that function’s regular documentation. For example, I could write the even? function this way:

```scheme
;;;; (even? n) is given a natural number n; it returns 1 if n is
;;;; even and 0 otherwise
;;;;
;;;; (define even? (n)
;;;;   (if (= n 0)
;;;;     1
;;;;     (if (= n 1)
;;;;       0
;;;;       (even? (- n 2)))))
;;;;
;;;; Recursion pattern follows the parity proof system from the handout
```

If the parity proof system didn’t appear in the handout, I might instead have written this:

```scheme
;;;; Recursion pattern follows this idea: 0 and 1 are natural
;;;; numbers. And if m is a natural number, so is m+2.
```

## A jumping-off point for your solution

You will put your solutions in a file solution.imp, and you will write up your whole assignment in a README file. At [http://www.cs.tufts.edu/comp/105/homework/solution_template.imp](http://www.cs.tufts.edu/comp/105/homework/solution_template.imp) and [http://www.cs.tufts.edu/comp/105/homework/README_template](http://www.cs.tufts.edu/comp/105/homework/README_template), we provide templates for solution.imp\(^5\) and README\(^6\).

To turn the solution template into a real solution, follow these steps for each function:

- In a comment, you will find a template for the function’s **contract**. Edit the contract to make it specific and accurate. Use the coding guidelines\(^7\).
- You will find one unit test that uses `check-error`. The test is a placeholder. Remove the `check-error` and replace it with `check-expect` unit tests, which you will write yourself. (You will need multiple unit tests: at least one per base case and per induction step.)
- You will find a function definition with the body `(/ 1 0)`. Evaluating this code divides 1 by 0, which causes an error. Replace this code with your implementation of the function.
- Below the function definition you will find a block comment containing a placeholder for your analysis of the recursion pattern in the function (either a proof system or an idea). Replace the placeholder with your actual analysis.
- If you write any helper functions, *supply each helper function with a contract and with unit tests.*

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\(^5\) solution_template.imp
\(^6\) README_template
\(^7\) ../coding-style.html#contracts
The problems you must solve

Do Exercises 4, 5, 7, 8, and 10 on pages 76–78 of Build, Prove, and Compare. Also do problems DD and O below.

DD. Define a function double-digit. When given a positive integer less than 20,000, double-digit returns a positive integer whose decimal representation is the same as the decimal representation of the input, except each digit appears twice. For example, (double-digit 123) is 112233.

O. Define a function overflow. Function overflow does not accept any arguments, and when called, it causes a checked run-time error indicating arithmetic overflow. In your submission, include a unit test that calls (overflow). The unit test must pass.

In problem 8, functions prime? and sumprimes expect nonnegative integers. Functions nthprime and relprime? expect positive integers.

The problems stress induction and recursion, which is the topic of the first class. And your recitation will address these kinds of problems. But a couple of them may still be challenging; if you have difficulty, consult a member of the course staff or ask questions on Piazza.

My solutions total 50–60 lines of Impcore.

Expectations for your solutions

This assignment lays the foundations for much that is to come. Here's what we expect:

- Your solutions must be valid Impcore; in particular, they must pass the following test:
  /comp/105/bin/impcore -q < solution.imp > /dev/null
  with no error messages and no unit-test failures. If your file produces error messages, we won’t test your solution and you will earn No Credit for functional correctness (you can still earn credit for readability).

- Your solutions must load and complete within 250 CPU milliseconds. If you write any long-running test, don’t include them in solution.imp; instead, create a file extra-tests.imp.

- On this assignment, as on several assignments to come, you must use recursion. Code using while loops will receive No Credit.

- You may use helper functions where appropriate, but you must not use global variables.

- Your code must be your own work. Do not share your solutions with anyone. We encourage you to discuss ideas, but no other student may see your code.

A photograph we can use to learn your name (10%)

I hope to learn the name of every student in the class. The teaching assistants and recitation leaders would also like to learn your name. But we need your help. Part of the assignment, for 10% of the grade on the assignment, is to submit a photograph that will help us learn to recognize you. I’ve consulted with a
skilled portrait photographer to prepare guidelines for producing a good, easily recognizable photograph, even if all you have is a cellphone camera. You’ll submit the photo as photo.jpg.

What to submit and how to submit it

You’ll choose a directory for your assignment, in which you will create or copy these files:

- **cqs.impcore.txt** will contain the comprehension questions and your answers
- **solution.imp** will contain your code, with its documentation and unit tests. Problems will appear in order by number, with problems DD and O last.
- **photo.jpg** will contain a recognizable photograph of you.
- **README** will
  - Identify anyone with whom you have collaborated or discussed the assignment
  - Carry any other information you wish to pass on to us

As soon as you have the files listed above, run `submit105-impcore` to submit a preliminary version of your work. The submit script checks your work and runs `provide` on your behalf. Do submit early, then keep submitting until your work is complete; we grade only the last submission.

The submit script will ask you some questions:

- Your preferred first and last names
- How we should pronounce your name, as in “kaeth-lEEEn FI-shur” or “NORE-muhn RAM-zee.”
- How many hours you worked on the assignment

You may also submit `extra-tests.imp`, which should contain only test code and unit tests (check-expect or check-error). You can run the tests using the Unix command

```
cat solution.imp extra-tests.imp | impcore -q
```

How your work will be evaluated

How your code will be evaluated

A big part of this assignment is for you to be sure you understand how we expect your code to be structured and organized. There is some material about this on the coding style page on the course website. When we get your work, we will evaluate it in two ways:

- About 50% of your grade for the assignment will be based on our judgement of the structure and organization of your code. To judge structure and organization, we will use the following dimensions:

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8../coding-style.html
– **Documentation** assesses whether your code is documented appropriately.

We expect you to document each function such that someone else could use your code and reason about its result without having to see the code itself. In particular, every function must be documented with a contract, and the contract must mention each parameter by name.

For this assignment, we also expect you to document your analysis of the inductive structure used by each function. Your analysis should explicitly identify which cases are base cases and which are inductive steps.

And remember that every named parameter should be mentioned in a function’s contract.

– **Unit testing** assesses whether your code is appropriately tested by some combination of check-expect, check-assert, and/or check-error. Appropriate testing exercises every part of your code.

When you are testing a recursive function based on a proof system from the handout, we expect a unit test for every proof rule whose conclusion has the form that something is a natural number. (For example, if your recursion pattern is based on the Decimal proof system, you need a test for the zero case and a test for the $10 \times n + d$ case. Do not include a test for each digit.)

When you are testing a recursive function whose recursion pattern is not based on a proof system from the handout, we expect a unit test for every idea you used about what constitutes a natural number.

– **Form** assesses whether your code uses indentation, line breaks, and comments in a way that makes it easy for us to read.

We expect you to use consistent indentation, to obey the offside rule\(^9\) described in the coding-style guidelines, and to limit the use of inline comments in the body of each function.

– **Naming** assesses your choice of names. To people who aspire to be great programmers, names matter deeply.

We give you a hand here by providing a template in which the names of top-level functions and their arguments are already chosen for you. For helper functions, you will choose your own names. Look at the general coding rubric\(^10\) and choose wisely.

– **Structure** assesses the underlying structure of your solution, not just how its elements are documented, formatted, and named.

We expect that if asked to use recursion, your solution will use recursion. Additionally, we expect that your recursive structure will be minimal, avoiding superfluous cases—especially redundant base cases—and unnecessary comparisons.

• About 30% of your grade for the assignment will be based on our judgement of the correctness of your code. We often look at code to see if it is correct, but our primary tool for assessing correctness is by testing. On a typical assignment, the correctness of your code would carry more weight, but relative to the other homeworks in 105, the problems on this assignment are very easy, so they carry less weight.

\(^9\)../coding-style.html#offside
\(^10\)../coding-rubric.html
The detailed criteria we will use to assess your code are found at http://www.cs.tufts.edu/comp/105/coding-rubric.html. Though things may be worded differently, most of these criteria are also applied in COMP 11, 15, and 40—they make explicit what we mean by “good programming practice.” But as you might imagine, there is a lot of information there—probably more than you can assimilate in one reading. The highlights are

- **Documentation**
  - Each function is documented with a contract that explains what the function returns, in terms of the parameters, which are mentioned by name. From documentation, it is easy to determine how each parameter affects the result.
  - The contract makes it possible to use the function without looking at the code in the body.
  - Documentation appears consistent with the code being described.
  - Each parameter is mentioned in the documentation at least once.
  - In every case analysis, all cases are necessary.

- **Unit Testing**
  - Unit tests cover the branches of execution for the function.
  - Unit tests test input which is valid per the problem definition.

- **Form**
  - Code fits in 80 columns.
  - Code respects the offside rule.
  - Code contains no tab characters.
  - Indentation is consistent everywhere.
  - If a construct spans multiple lines, its closing parenthesis appears at the end of a line, possibly grouped with one or more other closing parentheses—never on a line by itself.
  - No code is commented out.
  - Solutions load and run without calling `print`

- **Naming**
  - Each function is named either with a noun describing the result it returns, or with a verb describing the action it does to its argument. (Or the function is a predicate and is named as suggested below.)
  - A function that is used as a predicate (for if or while) has a name that is formed by writing a property followed by a question mark. Examples might include even? or prime?. (Applies only if the language permits question marks in names.)
  - In a function definition, the name of each parameter is a noun saying what, in the world of ideas, the parameter represents.
  - Or the name of a parameter is the name of an entity in the problem statement, or a name from the underlying mathematics.
  - Or the name of a parameter is short and conventional. For example, a magnitude or count might be n or m. An index might be i, j, or k. A pointer might be p; a string might be s. A variable might be x; an expression might be e.

- **Structure**
  - Solutions are recursive, as requested in the assignment.
  - There’s only as much code as is needed to do the job.
  - In the body of a recursive function, the code that handles the base case(s) appears before any recursive calls.
  - The code of each function is so clear that, with the help of the function’s contract, course staff can easily tell whether the code is correct or incorrect.

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Expressions cannot be made any simpler by application of algebraic laws.

How your photograph will be evaluated

If your photograph is clear, makes it easy to recognize you, and is not ridiculous in size, it will earn a grade of Very Good (the top grade). If you’re not sure how to take a photograph that makes you easy to recognize, I consulted with a skilled portrait photographer for some tips. In the table below, you want to aim for “Exemplary” (the left column), be willing to settle for “Satisfactory” (the middle column), and avoid “Must Improve” (the right column).\(^\text{12}\)

<table>
<thead>
<tr>
<th>Exemplary</th>
<th>Satisfactory</th>
<th>Must Improve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head and shoulders fill 2/3 to 3/4 of the frame.</td>
<td>Face fills the frame; shoulders not visible.</td>
<td>Photo down to waist; full-body photo.</td>
</tr>
<tr>
<td>The shot is taken from a distance of 4 to 6 feet, and the camera is zoomed as needed to include just head and shoulders.</td>
<td>The subject is not looking at the camera, but there is a normal look on the face.</td>
<td>More than one person visible in photo.</td>
</tr>
<tr>
<td>Or, the shot is taken from a distance of 4 to 6 feet, then cropped to include just head and shoulders.</td>
<td></td>
<td>Eyes are closed.</td>
</tr>
<tr>
<td>The subject is looking at the camera with a normal look on the face.</td>
<td></td>
<td>The camera is too close to the subject. (This will happen if you compose the shot by moving the camera toward the subject until the subject’s head and shoulders fill the “viewfinder”; you’ll get perspective distortion.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The subject is making a strange face (eye rolls, etc)</td>
</tr>
<tr>
<td></td>
<td>The subject’s face is lit evenly.</td>
<td>The background is significantly brighter than the subject.</td>
</tr>
<tr>
<td></td>
<td>The subject’s face is lit by ambient light, plus flash bounced off a ceiling or wall (possible only with a real camera)</td>
<td>There is light behind the subject aimed at the camera.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The sun is shining in the subject’s face.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The subject is illuminated by a camera flash.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{12}\) Yes, a student once sent me a photograph of two people.
<table>
<thead>
<tr>
<th>Focus</th>
<th>Exemplary</th>
<th>Satisfactory</th>
<th>Must Improve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• The subject is in sharp focus, while the</td>
<td>• Some part of the subject is in sharp focus, or</td>
<td>• The subject’s face is out of focus.</td>
</tr>
<tr>
<td></td>
<td>background is a little blurry (possibly only</td>
<td>something near the subject is in sharp focus.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>with a real camera or with very sophisticated</td>
<td>The subject’s face is not in sharp focus but is</td>
<td></td>
</tr>
<tr>
<td></td>
<td>software).</td>
<td>still easy to recognize.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• The subject’s face is in sharp focus.</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>• Some part of the subject is in sharp focus,</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>or something near the subject is in sharp</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>focus. The subject’s face is not in sharp</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>focus but is still easy to recognize.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• The subject's face is out of focus.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>File</th>
<th>Exemplary</th>
<th>Satisfactory</th>
<th>Must Improve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• The uploaded image file is from 300KB to 1.2MB in size.</td>
<td>• The uploaded image file is at most 2MB in size.</td>
<td>• The uploaded image file is more than 2MB in size.</td>
</tr>
<tr>
<td></td>
<td>• Resolution of the uploaded file is high</td>
<td>• When shown at a few inches high, the uploaded</td>
<td>• At its natural resolution, the uploaded image file shows pixels or compression artifacts.</td>
</tr>
<tr>
<td></td>
<td>enough that there’s no pixelation.</td>
<td>image file is pixelated or has compression blur.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• The uploaded image is at least as tall as it is wide (portrait orientation)</td>
<td></td>
<td>• The uploaded image is wider than it is tall (landscape orientation)</td>
</tr>
</tbody>
</table>