Using polymorphic names

\[ \rightarrow (\text{val } \text{cc} \ (\text{lambda} \ (\text{nss}) \ (\text{car} \ (\text{car} \ \text{nss})))) \]
Using polymorphic names

\[
\rightarrow (\text{val cc (lambda (nss) (car (car nss))))}
\]

\[
\text{cc : (forall ('a) ((list (list 'a)) \rightarrow 'a))}
\]
Refresh your skills!

-> (val second (lambda (xs) (car (cdr xs)))))
second : ...

-> (val two (lambda (f) (lambda (x) (f (f x))))))
two : ...
Skills refreshed

-> (val second (lambda (xs) (car (cdr xs))))
second : (forall ('a) ((list 'a) -> 'a))
-> (val two (lambda (f) (lambda (x) (f (f x)))))
two : (forall ('a) (('a -> 'a) -> ('a -> 'a)))
Making Type Inference Precise

Sad news:
- Type inference for polymorphism is undecidable

Solution:
- Each formal parameter has a monomorphic type

Consequences:
- The *argument* to a higher-order function *cannot* be polymorphic
- *forall* appears only outermost in types
We infer stratified "Hindley-Milner" types

Two layers: Monomorphic types $\tau$
Polymorphic type schemes $\sigma$

$\tau ::= \alpha$ type variables
$\mid \mu$ type constructors: int, list
$\mid (\tau_1, \ldots, \tau_n) \tau$ constructor application

$\sigma ::= \forall \alpha_1, \ldots, \alpha_n . \tau$ type scheme

Each variable in $\Gamma$ introduced via LET, LETREC, VAL, and VAL-REC has a type scheme $\sigma$ with $\forall$

Each variable in $\Gamma$ introduced via LAMBDA has a degenerate type scheme $\forall . \tau$—a type, wrapped
Representing Hindley-Milner types

datatype ty
    = TYCON of name
    | CONAPP of ty * ty list
    | TYVAR of name

datatype type_scheme
    = FORALL of name list * ty

fun funtype (args, result) =
    CONAPP (TYCON "function",
            [CONAPP (TYCON "arguments", args),
             result])
Key ideas

Type environment $\Gamma$ binds var to type scheme $\sigma$

- $\text{app2} : \forall \alpha, \beta. (\alpha \to \beta) \times \alpha \times \alpha \to \beta$
- $\text{cc} : \forall \alpha. \alpha \text{ list list} \to \alpha$
- $\text{car} : \forall \alpha. \alpha \text{ list} \to \alpha$
- $\text{n} : \forall. \text{int} \quad \text{(note empty $\forall$)}$

Judgment $\Gamma \vdash e : \tau$ gives expression $e$ a type $\tau$

(Transitions happen automatically!)
Key ideas

Definitions are polymorphic with type schemes

Each use is monomorphic with a (mono-) type

Transitions:
• At use, type scheme instantiated automatically
• At definition, automatically abstract over tyvars
All the pieces

1. Hindley-Milner types
2. Bound names: $\sigma$, expressions: $\tau$
3. Type inference yields type-equality constraint
4. Constraint solving produces substitution
5. Substitution refines types
6. Call solver, introduce polytypes at `val`
7. Call solver, introduce polytypes at all `let forms`
Type-inference algorithm

Given $\Gamma$ and $e$, compute $C$ and $\tau$ such that

$$C, \Gamma \vdash e : \tau$$

Idea #2: Extend to list of $e_i$: $C, \Gamma \vdash e_1, \ldots, e_n : \tau_1, \ldots, \tau_n$

$$\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau$$

$$\Gamma \vdash \text{IF}(e_1, e_2, e_3) : \tau$$

becomes (note equality constraints with $\sim$)

$$C, \Gamma \vdash e_1, e_2, e_3 : \tau_1, \tau_2, \tau_3$$

$$C \land \tau_1 \sim \text{bool} \land \tau_2 \sim \tau_3, \Gamma \vdash \text{IF}(e_1, e_2, e_3) : \tau_3$$

(IF)
Apply rule

\[\Gamma \vdash e : \tau_1 \times \cdots \times \tau_n \rightarrow \tau \quad \Gamma \vdash e_1 : \tau_1 \quad \cdots \quad \Gamma \vdash e_n : \tau_n\]

\[\Gamma \vdash \text{APPLY}(e, e_1, \ldots, e_n) : \tau\]  

\[(\text{APPLY})\]

becomes

\[C, \Gamma \vdash e, e_1, \ldots, e_n : \tau_f, \tau_1, \ldots, \tau_n \quad \alpha \text{ is fresh}\]

\[C \land \tau_f \sim \tau_1 \times \cdots \times \tau_n \rightarrow \alpha, \Gamma \vdash \text{APPLY}(e, e_1, \ldots, e_n) : \alpha\]  

\[(\text{APPLY})\]
Your turn: Begin Rule

\[
\Gamma \vdash e_i : \tau_i \quad 1 \leq i \leq n
\]

\[
\Gamma \vdash \text{BEGIN}(e_1, \ldots, e_n) : \tau_n
\]

\[\text{(BEGIN)}\]

\[
C, \Gamma \vdash e_1, \ldots, e_n : \tau_1, \ldots, \tau_n
\]

\[
C, \Gamma \vdash \text{BEGIN}(e_1, \ldots, e_n) : \tau_n
\]

\[\text{(BEGIN)}\]
Type inference, operationally

Like type checking:
- Top-down, bottom up pass over abstract syntax
- Use $\Gamma$ to look up types of variables

Different from type checking:
- Create fresh type variables when needed
- Accumulate equality constraints
Your skills so far

You can complete `typeof`
  - Takes $e$ and $\Gamma$, returns $\tau$ and $C$

(Except for `let` forms.)

Next up: solving constraints!