Evaluation does not add or remove a global variable

For any $e$, $\xi$, $\phi$, $\rho$, $v$, $\xi'$, and $\rho'$ such that

$$\langle e, \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle,$$

we can prove

$$\text{dom } \xi = \text{dom } \xi'$$

Really means: “Both sides have the same domain”
Assume the existence of a derivation

Could terminate in any rule!

Base case:

\[ \langle \text{LITERAL}(v), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho \rangle \]

Both sides identical!

\[ \text{dom } \xi = \text{dom } \xi \]
Holds for formal-parameter lookup

Another base case:

\[
x \in \text{dom } \rho \quad \text{\underline{\Rightarrow}} \quad \langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle
\]

Both sides identical!

\[
\text{dom } \xi = \text{dom } \xi
\]
Inductive case: good sub-derivation

Assignment to formal parameter

\[
\begin{align*}
\frac{x \in \text{dom } \rho \quad \langle e, \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle \text{SET}(x,e), \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \{x \mapsto v\} \rangle}
\end{align*}
\]

By induction hypothesis on \( \mathcal{D} \), \( \text{dom } \xi = \text{dom } \xi' \)

Both sides have same domain!
Inductive case: good sub-derivation

True conditional

<table>
<thead>
<tr>
<th>(e_1, \xi, \phi, \rho)</th>
<th>(v_1, \xi', \phi, \rho')</th>
<th>(v_1 \neq 0)</th>
<th>(e_2, \xi', \phi, \rho')</th>
<th>(v_2, \xi'', \phi, \rho'')</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{IF}(e_1, e_2, e_3), \xi, \phi, \rho)</td>
<td>(v_2, \xi'', \phi, \rho'')</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

By induction hypothesis on \(\mathcal{D}_1\), \(\text{dom} \xi = \text{dom} \xi'\)

By induction hypothesis on \(\mathcal{D}_2\), \(\text{dom} \xi' = \text{dom} \xi''\)

Therefore, both sides have same domain:
\(\text{dom} \xi = \text{dom} \xi''\)
The only interesting case: assign to global

\[ \frac{x \notin \text{dom } \rho \quad x \in \text{dom } \xi}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle} \]

Do both sides have same domain?

- **Does** \( \text{dom } \xi = \text{dom}(\xi' \{ x \mapsto v \}) \) ?

By induction hypothesis on \( \mathcal{D} \), \( \text{dom } \xi = \text{dom } \xi' \)

And \( \text{dom}(\xi' \{ x \mapsto v \}) = \text{dom } \xi' \cup \{ x \} = \text{dom } \xi \cup \{ x \} \)

But \( x \in \text{dom } \xi! \) So \( \text{dom } \xi \cup \{ x \} = \text{dom } \xi \)
And now, Scheme